Robust H_∞ Control for Satellite Attitude Control System with Uncertainties and Additive Perturbation

Chuang Liu[1,a], Feng Wang[*1,2,b], Keke Shi[1,c],Xuefeng Wang[1,d], Zhaowei Sun[1]

[1]Research Center of Satellite Technology, Harbin Institute of Technology, Harbin 150001, China

[2]National&Local United Engineering Research Center of Small Satellite Technology, Changchun 130033, China

[a]liuchuangforever@msn.com, [b]wfhitsat@hit.edu.cn [c]920205365@qq.com, [d]wangxuefeng2324@qq.com

Abstract. This paper proposed a robust non-fragile H_∞ control method for satellite attitude control system with uncertainties and subject to external disturbances, gyro drifts and controller perturbation. First, the model of satellite attitude system is established and converted into a state space form with uncertainties. Sufficient condition for the existence of state feedback robust non-fragile H_∞ controller in terms of additive perturbation is given based on linear matrix inequalities (LMIs). When a feasible solution of LMIs is obtained, the controller gain can be known under the condition of certain given values. Then, the theorem of mixed H_2/H_∞ is given to compare with the method proposed in this paper. The simulation results of non-fragile H_∞ control and mixed H_2/H_∞ based on closed-loop satellite attitude control system are presented to demonstrate the effectiveness and performance of the proposed control method.

Keywords: Satellite attitude control system; Parameter uncertainties; Additive perturbation; H_∞ control.

1. Introduction

Many uncertain factors such as external disturbance torques, gyro drifts and controller perturbations may result in damage and affect the satellite performance during its on-orbit service. Satellite attitude control system becomes more and more sophisticated due to the increasing requirements for high functionality and performance. In the bad space environment, the complex system inevitably confronts different types of disturbance. Methods centered on robust H_∞ control have attracted considerable attention during the past few years. Robust non-fragile control has been widely applied in neutral dynamic systems [1], stochastic nonlinear time-delay systems [2], active magnetic bearing system [3] and so on.

The robust H_∞ controller can guarantee that the closed-loop system still satisfies certain H_∞ performance when bounded parameter uncertainties exist [4,5]. Nevertheless, the performance of robustness also relies on precise realization of controller. The controller parameters could accrue some variations due to some disturbances. The traditional feedback control methods are sensitive to the small variations [6]. It has shown that even a very small perturbation on controller parameters may lead to the degradation of performance or destabilize the closed-loop system. Sensitivity analysis of H_∞ quadratic stability problem for continuous-time system was performed to show that proper method can lead to tight perturbation bounds [7]. Thus, it is necessary to design a controller that is robust against its own parameters variations which is called non-fragile controller. This problem has been widely investigated by many researchers, who have applied control theory to non-fragile control problem with different requirements. For example, a non-fragile controller was designed by solving a pair of indefinite algebraic Riccati equations for a known linear time-invariant system in [8]. A non-fragile procedure was introduced in [9] to study the problem of synchronization of neural networks with time-varying delay. However, due to the complicated satellite attitude control system, the parameter uncertainties have not

[*] Corresponding author of this paper

been all taken into consideration in the existing studies. Moreover, they have less relation with satellite attitude control problems in the above literatures.

The main contribution of this paper is to propose the robust non-fragile H_∞ control method for a class of satellite attitude control systems with parameter uncertainties and subject to external disturbance torques, gyro drifts, controller perturbations, H_∞ performance constraint and quadratic stability. Based on the Lyapunov theory, sufficient conditions for the existence of robust non-fragile state feedback controller are given based on linear matrix inequalities (LMIs). Then, the robust non-fragile H_∞ state feedback controller can be regarded as a convex optimization problem subject to LMI constraints. Once the controller is obtained, the satellite attitude maneuver and stability can be accomplished. The simulation results based on satellite attitude control system are presented to demonstrate the effectiveness of the proposed control method.

2. Satellite Attitude Dynamics

Consider a rigid-body satellite in a circular orbit, the three coordinate frames for satellite attitude dynamics modeling include the local vertical and local horizontal reference frame (LVLH) with its origin at the center of mass of the satellite, a satellite centered satellite fixed reference frame (SCSF) and an earth centered inertial reference frame (ECI).

In the inertial coordinate system, the attitude dynamics of a satellite can be described as

$$I_b\dot{\omega} + \omega \times (I_b\omega) = T_c + T_g + T_d \qquad (1)$$

Choose

$$I_b = diag(I_x, I_y, I_z)$$

And let

$$T_c = \begin{bmatrix} T_{cx} \\ T_{cy} \\ T_{cz} \end{bmatrix}, T_g = \begin{bmatrix} T_{gx} \\ T_{gy} \\ T_{gz} \end{bmatrix}, T_d = \begin{bmatrix} T_{dx} \\ T_{dy} \\ T_{dz} \end{bmatrix}$$

Then, equation (1) can be converted into

$$\begin{cases} I_x\dot{\omega}_x + (I_z - I_y)\omega_y\omega_z = T_{cx} + T_{gx} + T_{dx} \\ I_y\dot{\omega}_y + (I_x - I_z)\omega_z\omega_x = T_{cy} + T_{gy} + T_{dy} \\ I_z\dot{\omega}_z + (I_y - I_x)\omega_x\omega_y = T_{cz} + T_{gz} + T_{dz} \end{cases} \qquad (2)$$

To describe the orientation of SCSF with respect to LVLH, in terms of three Euler angles φ, θ and ψ, which are roll, pitch and yaw attitude angles respectively, a typical sequence of three successive body-axis rotations is yaw→pitch→roll.

Under small angle approximation, we can get

$$\begin{cases} I_x\ddot{\varphi} + \omega_0^2(I_y - I_z)\varphi + (I_y - I_z - I_x)\omega_0\dot{\psi} = T_{cx} + T_{gx} + T_{dx} \\ I_x\ddot{\theta} = T_{cy} + T_{gy} + T_{dy} \\ I_z\ddot{\psi} + \omega_0^2(I_y - I_x)\dot{\psi} + (I_x + I_z - I_y)\omega_0\dot{\varphi} = T_{cz} + T_{gz} + T_{dz} \end{cases} \qquad (3)$$

As is known, Tg is easily shown as

$$T_g = \begin{bmatrix} T_{gx} \\ T_{gy} \\ T_{gz} \end{bmatrix} = \begin{bmatrix} -3\omega_0^2 (I_y - I_z)\varphi \\ -3\omega_0^2 (I_x - I_z)\theta \\ 0 \end{bmatrix} \qquad (4)$$

Then, the final attitude dynamic equation can be obtained

$$\begin{cases} I_x\ddot{\varphi} + 4(I_y - I_z)\omega_0^2\varphi + (I_y - I_z - I_x)\omega_0\dot{\psi} = T_{cx} + T_{dx} \\ I_y\ddot{\theta} + 3\omega_0^2 (I_x - I_z)\theta = T_{cy} + T_{dy} \\ I_z\ddot{\psi} + (I_y - I_x)\omega_0^2\psi + (I_x + I_z - I_y)\omega_0\dot{\varphi} = T_{cz} + T_{dz} \end{cases} \qquad (5)$$

Choosing state variable as

$$x = [\varphi \quad \theta \quad \psi \quad \dot{\varphi} \quad \dot{\theta} \quad \dot{\psi}]^T, y = [\varphi \quad \theta \quad \psi \quad \omega_x \quad \omega_y \quad \omega_z]^T$$

The satellite attitude control system model with gyro drift can be obtained as follows:

$$\begin{cases} \dot{x}(\mathrm{t}) = (A + \Delta A)x(\mathrm{t}) + B_1 u(\mathrm{t}) + B_2 w(\mathrm{t}) \\ y(\mathrm{t}) = C_1 x(\mathrm{t}) + D_1 d(\mathrm{t}) + \mathrm{D}_2 \\ z(\mathrm{t}) = C_2 x(\mathrm{t}) \end{cases} \qquad (6)$$

where $x(t)$ is the vector of state variables, u(t) is the vector of control inputs, w(t) is the vector of disturbance inputs, d(t) is the vector of gyro drift, y(t) and z(t) are the measurable output and controlled output, $\Delta A(\mathrm{t})$ is the parameter uncertainty which is the form of (7), others are system coefficient matrices of appropriate dimensions.

$$\Delta A(t) = M_1 F_1(t) N_1, F_1(t)^T F_1(t) \le I, \ \forall t \qquad (7)$$

where M_1 and N_1 are known constant real matrices of appropriate dimension, $F_1(t)$ is an unknown matrix function with Lebesgue measurable elements.

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -4\omega_0^2 I_x^{-1}(I_y - I_z) & 0 & 0 & 0 & 0 & -\omega_0 I_x^{-1}(I_y - I_x - I_z) \\ 0 & -3\omega_0^2 I_y^{-1}(I_x - I_z) & 0 & 0 & 0 & 0 \\ 0 & 0 & -\omega_0^2 I_z^{-1}(I_y - I_x) & \omega_0 I_z^{-1}(I_y - I_x - I_z) & 0 & 0 \end{bmatrix},$$

$B_1 = B_2 = D_1 = \left[0_{3\times3} \quad diag(\mathrm{I}_x^{-1}, \mathrm{I}_y^{-1}, \mathrm{I}_z^{-1})\right]$, $u = \left[T_{cx} \quad T_{cy} \quad T_{cz}\right]^T$, $w = \left[T_{dx} \quad T_{dy} \quad T_{dz}\right]^T$,

$D_2 = \left[0_{1\times4} \quad -\omega_0 \quad 0\right]$, $C_1 = \begin{bmatrix} I_{3\times3} & 0_{3\times3} \\ B & I_{3\times3} \end{bmatrix}$, $C_2 = \left[I_{6\times6}\right]$.

3. The Design of Controller

Design a state feedback robust non-fragile H∞ controller

$$u(\mathrm{t}) = (K + \Delta K)x(\mathrm{t}) \qquad (8)$$

ΔK means controller gain perturbation, here only considers additive perturbation, so

$$\Delta K = M_2 F_2(t) N_2, \ F_2(t)^T F_2(t) \le I \qquad (9)$$

Substituting (8) into (6) yields the following closed-loop system:

$$\begin{cases} \dot{x}(t) = \left(A + \Delta A + B_1 K + B_1 \Delta K\right) x + B_2 w(t) \\ z(t) = C_2 x(t) \end{cases} \quad (10)$$

For all uncertainties ΔA(t) and ΔK， the following two conditions are satisfied.

（1） Closed-loop system (10) is quadratically stable.

（2） For a given $\gamma > 0$, z(t) satisfies H∞ performance constraint.

Definition 1 If there is a symmetric positive definite matrix P>0 and a positive constant α , for arbitrary uncertainty, the time derivative of Lyapunov function $V(\mathrm{x}, \mathrm{t})$ satisfies

$$\dot{V}(\mathrm{x}, \mathrm{t}) \le -\alpha \left\| x(\mathrm{t}) \right\|_2^2 \quad (11)$$

Then, system (10) ($w(\mathrm{t}) = 0$) is quadratically stable.

Definition 2 For a given $\gamma > 0$, if the controlled output z(t) satisfies (11), then, we can say z(t) satisfies H_∞ performance constraint.

$$\left\| z(\mathrm{t}) \right\|_2^2 < \gamma^2 \left\| w(\mathrm{t}) \right\|_2^2 \quad (12)$$

Lemma 1 Let $H, E \in R^{m \times n}, F(t) \in S^m$, and $F(t)$ satisfies $F(\mathrm{t})^T F(\mathrm{t}) \le \mathrm{I}$, then for a scalar $\xi > 0$,

$$HF(\mathrm{t})E + E^T F(\mathrm{t})^T H^T \le \xi^{-1} H H^T + \xi E^T E \quad (13)$$

Lemma 2 (Schur complement lemma) Let the partitioned matrix

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{12}^T & A_{22} \end{bmatrix} \quad (14)$$

be symmetric. Then

$$A < 0 \Leftrightarrow A_{11} < 0, A_{22} - A_{12}^T A_{11}^{-1} A_{12} < 0 \Leftrightarrow A_{22} < 0, A_{11} - A_{12} A_{22}^{-1} A_{12}^T < 0 \quad (15)$$

or

$$A > 0 \Leftrightarrow A_{11} > 0, A_{22} - A_{12}^T A_{11}^{-1} A_{12} > 0 \Leftrightarrow A_{22} > 0, A_{11} - A_{12} A_{22}^{-1} A_{12}^T > 0 \quad (16)$$

Theorem1 When controller gain perturbation is the form of (9), for given $\xi_1 > 0, \xi_2 > 0$ and $\gamma > 0$,uncertain system (10) is quadratically stable under the condition of robust non-fragile H∞ controller (8), and z(t) satisfies H∞ performance constraint if there exists a symmetric positive definite matrix X and a matrix W, such that the following LMI holds:

$$\begin{bmatrix} AX + B_1 W + XA^T + W^T B_1^T & M_1 & XN_1^T & B_1 M_2 & XN_2^T & XC_2^T & B_2 \\ M_1^T & -\xi_1^{-1} I & 0 & 0 & 0 & 0 & 0 \\ N_1 X & 0 & -\xi_1 I & 0 & 0 & 0 & 0 \\ (B_1 M_2)^\mathrm{T} & 0 & 0 & -\xi_2^{-1} I & 0 & 0 & 0 \\ N_2 X & 0 & 0 & 0 & -\xi_2 I & 0 & 0 \\ C_2 X & 0 & 0 & 0 & 0 & -I & 0 \\ B_2^T & 0 & 0 & 0 & 0 & 0 & -\gamma^2 I \end{bmatrix} < 0 \quad (17)$$

Furthermore, if LMI (17) has a feasible solution, the state feedback controller $K = WX^{-1}$.

Proof Define Lyapunov function

$$V(\mathrm{x}(\mathrm{t})) = \mathrm{x}(\mathrm{t})^T P x(\mathrm{t})$$

Where $P = X^{-1}$.

Then

$$
\begin{aligned}
\dot{V}(\mathrm{x(t)}) &= \dot{\mathrm{x}}(\mathrm{t})^T Px(\mathrm{t}) + \mathrm{x(t)}^T P\dot{x}(\mathrm{t}) \\
&= \mathrm{x(t)}^T (A+\Delta A + B_1K + B_1\Delta K)^T Px(\mathrm{t}) + \mathrm{x(t)}^T P(A+\Delta A + B_1K + B_1\Delta K)\,\mathrm{x(t)} \\
&= \mathrm{x(t)}^T (A+M_1F_1(\mathrm{t})\mathrm{N}_1 + B_1K + B_1M_2F_2(\mathrm{t})N_2)^T Px(\mathrm{t}) + \mathrm{x(t)}^T P(A+M_1F_1(\mathrm{t})N_1 + B_1K + B_1M_2F_2(\mathrm{t})N_2)\,\mathrm{x(t)} \\
&= \mathrm{x(t)}^T \left[(A+B_1K)^T + (M_1F_1(\mathrm{t})\mathrm{N}_1 + B_1M_2F_2(\mathrm{t})N_2)^T\right] Px(\mathrm{t}) + \mathrm{x(t)}^T P\left[(A+B_1K) + (M_1F_1(\mathrm{t})N_1 + B_1M_2F_2(\mathrm{t})N_2)\right]\mathrm{x(t)} \\
&= \mathrm{x(t)}^T \left[(A+B_1K)^T P + (M_1F_1(\mathrm{t})\mathrm{N}_1 + B_1M_2F_2(\mathrm{t})N_2)^T P + P(A+B_1K) + P(M_1F_1(\mathrm{t})N_1 + B_1M_2F_2(\mathrm{t})N_2)\right] x(\mathrm{t}) \\
&= \mathrm{x(t)}^T \left[(A+B_1K)^T P + P(A+B_1K) + PM_1F_1(\mathrm{t})N_1 + \mathrm{N}_1^T F_1(\mathrm{t})^T M_1^T P + PB_1M_2F_2(\mathrm{t})N_2 + N_2^T F_2(\mathrm{t})^T M_2^T B_1^T P\right] x(\mathrm{t}) \\
&\le \mathrm{x(t)}^T \left[(A+B_1K)^T P + P(A+B_1K) + \xi_1 PM_1M_1^T P + \xi_1^{-1}\mathrm{N}_1^T N_1 + \xi_2 PB_1M_2M_2^T B_1^T P + \xi_2^{-1} N_2^T N_2\right] x(\mathrm{t})
\end{aligned}
$$

Considering(17), multiplied by $diag\{P,\ I,\ I,\ I,\ I,\ I,\ I\}$ at both sides simultaneously, (18) can be obtained.

$$
\begin{bmatrix}
PA + PB_1K + A^T P + K^T B_1^T P & PM_1 & N_1^T & PB_1M_2 & N_2^T & C_2^T & PB_2 \\
M_1^T P & -\xi_1^{-1} I & 0 & 0 & 0 & 0 & 0 \\
N_1 & 0 & -\xi_1 I & 0 & 0 & 0 & 0 \\
(B_1M_2)^{\mathrm{T}} P & 0 & 0 & -\xi_2^{-1} I & 0 & 0 & 0 \\
N_2 & 0 & 0 & 0 & -\xi_2 I & 0 & 0 \\
C_2 & 0 & 0 & 0 & 0 & -I & 0 \\
B_2^T P & 0 & 0 & 0 & 0 & 0 & -\gamma^2 I
\end{bmatrix} < 0 \qquad (18)
$$

Let

$$M_0 = (A+B_1K)^T P + P(A+B_1K) + \xi_1 PM_1M_1^T P + \xi_1^{-1}\mathrm{N}_1^T N_1 + \xi_2 PB_1M_2M_2^T B_1^T P + \xi_2^{-1} N_2^T N_2$$

According to(18), we can know $M_0 < 0$

Furthermore,

$$\dot{V}(\mathrm{x(t)}) \le \mathrm{x(t)}^T M_0\ \mathrm{x(t)} \le \lambda_{\max}(M_0)\,\mathrm{x(t)}^T\ \mathrm{x(t)}$$

Let $\alpha = -\lambda_{\max}(M_0) > 0$, then,

$$\dot{V}(\mathrm{x,t}) \le -\alpha \left\| x(\mathrm{t}) \right\|_2^2$$

By **definition 1**, system (10) is quadratically stable under the condition of robust non-fragile H_∞ controller (8).

To establish the $L_2\,[0, \infty)$ norm bound $\gamma^2 \left\| w(\mathrm{t}) \right\|_2^2$, consider the following functional:

$$J = \int_0^{+\infty} [z(\mathrm{t})^T z(\mathrm{t}) - \gamma^2 w(\mathrm{t})^T w(\mathrm{t})]dt$$

As closed-loop system has quadratic stability, for arbitrary nonzero $w(\mathrm{t}) \in \mathrm{L}_2[0,\infty)$, let x (0) =0, then,

$$
\begin{aligned}
J &= \int_0^{+\infty} [z(\mathrm{t})^T z(\mathrm{t}) - \gamma^2 w(\mathrm{t})^T w(\mathrm{t}) + \dot{V}(x(t))]dt - V(\mathrm{x}(\infty)) + V(0) \\
&\le \int_0^{\infty} \{x(\mathrm{t})^T C_2^T C_2 x(\mathrm{t}) - \gamma^2 w(\mathrm{t})^T w(\mathrm{t}) + x(\mathrm{t})^T [(A+B_1K)^T P + P(A+B_1K) + \xi_1 PM_1M_1^T P \\
&\quad + \xi_1^{-1}\mathrm{N}_1^T N_1 + \xi_2 PB_1M_2M_2^T B_1^T P + \xi_2^{-1} N_2^T N_2] x(\mathrm{t}) + w(\mathrm{t})^T B_2^T Px(\mathrm{t}) + x(\mathrm{t})^T PB_2 w(\mathrm{t})\}\,\mathrm{dt} \\
&\le \int_0^{\infty} \begin{bmatrix} x(\mathrm{t})^T & w(\mathrm{t})^T \end{bmatrix}
\begin{bmatrix}
\begin{matrix}(A+B_1K)^T P + P(A+B_1K) + \xi_1 PM_1M_1^T P \\ +\xi_1^{-1}\mathrm{N}_1^T N_1 + \xi_2 PB_1M_2M_2^T B_1^T P + \xi_2^{-1} N_2^T N_2 + C_2^T C_2\end{matrix} & PB_2 \\
B_2^T P & -\gamma^2 I
\end{bmatrix}
\begin{bmatrix} x(\mathrm{t}) \\ w(\mathrm{t}) \end{bmatrix}
\end{aligned}
$$

According to schur complement lemma and (18), $J < 0$ holds, namely, z(t) satisfies H_∞ performance constraint.

Theorem 2[10] . The performance of H_2 and H_∞ are both satisfied if and only if there exist W , two symmetric matrices Z and X such that

$$\begin{cases} AX + B_1W + (AX + B_1W)^T + B_2B_2^T < 0 \\ \begin{bmatrix} -Z & C_2X \\ (C_2X)^T & -X \end{bmatrix} < 0 \\ trace(\mathrm{Z}) < \rho \\ \begin{bmatrix} AX + B_1W + (AX + B_1W)^T & B_2 & (C_1X + D_1W)^T \\ B_2^T & -\gamma_\infty I & D_2^T \\ C_1X + D_1W & D_2 & -\gamma_\infty I \end{bmatrix} < 0 \end{cases} \quad (19)$$

By minimizing $c_\infty\gamma_\infty + c_2\rho$, where $\rho = \gamma_2^2$, the state feedback gain matrix can be constructed as K=WX^{-1}.

4. Numerical Results

In this section, non-fragile H_∞ control approach and mixed H_2/H_∞ method are illustrated with simulations on a satellite attitude control system with uncertainties to demonstrate the effectiveness of the proposed method.

The inertia parameters are assumed as $I_x = 200kg \cdot m^2, I_y = 200kg \cdot m^2, I_z = 30kg \cdot m^2$, the orbit height is 300km. Attitude angles and angular velocities are measured with star sensors and gyros. The constant value of gyro drift [11] can be assumed as d(t)=6 ×[10^{-5} 10^{-5} 10^{-5}]^T.

Choose the initial state as

$$x(0) = [0.07\ 0.06\ 0.05\ 0.012\ 0.010\ 0.008]^T$$

Where, the unit of angle is rad and the unit of angular velocity is rad/s. The expected state is zero. The disturbances are white noise modeled as

$$w = \begin{bmatrix} 0.5\cos(10^5\omega t) \\ 0.5\cos(10^5\omega t + \pi / 4) \\ 0.5\cos(10^5\omega t + \pi / 3) \end{bmatrix} N \cdot m$$

Meanwhile, choose

M_1=[0.8 1.1 1.3 1.5 1.6 1.8]^T , N_1=[-0.1 -0.2 -0.3 -0.4 -0.2 1],

M_2=[0.01 0.01 0.01]^T , N_2=[0.1 0.01 0.1 0.01 0.1 0.01],

ξ_1=0.1,ξ_2=0.1,γ=0.1， $F_1(\mathrm{t}) = 0.5\sin(\omega_0\ \mathrm{t}), F_2(\mathrm{t}) = 0.5\sin(\omega_0\ \mathrm{t} + \pi / 4)$,

With **Theorem1** we can attain the corresponding state feedback gain as follow

$$K = \begin{bmatrix} -6403.48 & -2918.61 & -18241.85 & -6377.88 & -997.95 & 3162.98 \\ 2936.43 & -12946.14 & -30455.17 & -1002.68 & -5872.17 & 1278.85 \\ 581.26 & -842.72 & -6355.15 & 525.36 & 169.28 & -2270.24 \end{bmatrix}$$

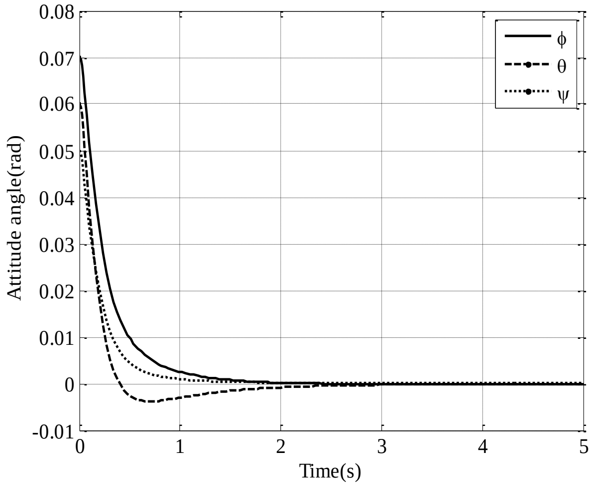

FIG. 1 Attitude angle

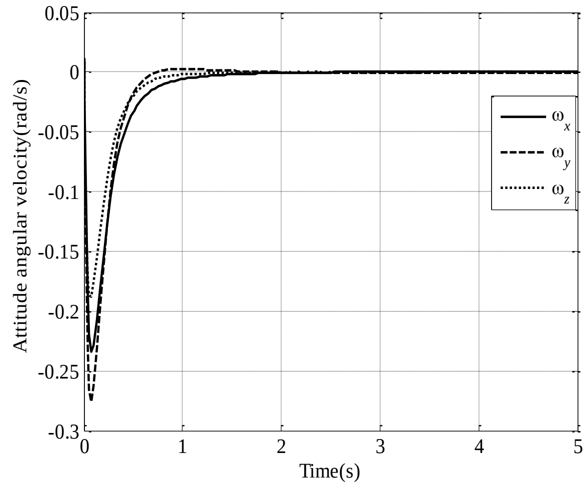

FIG. 2 Angular velocity

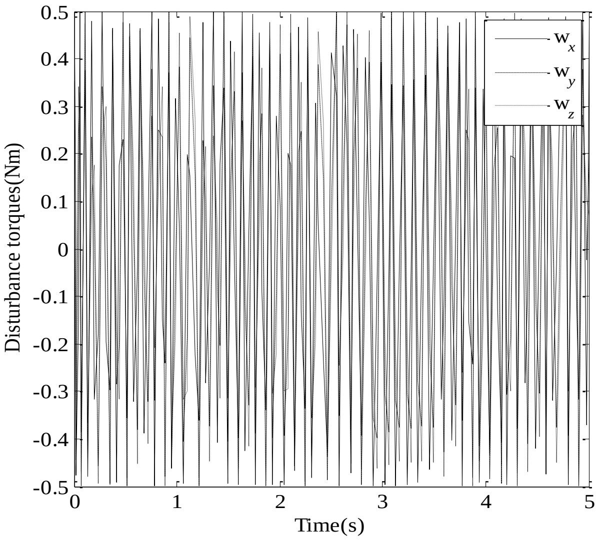

FIG. 3 Disturbance torques of white noise

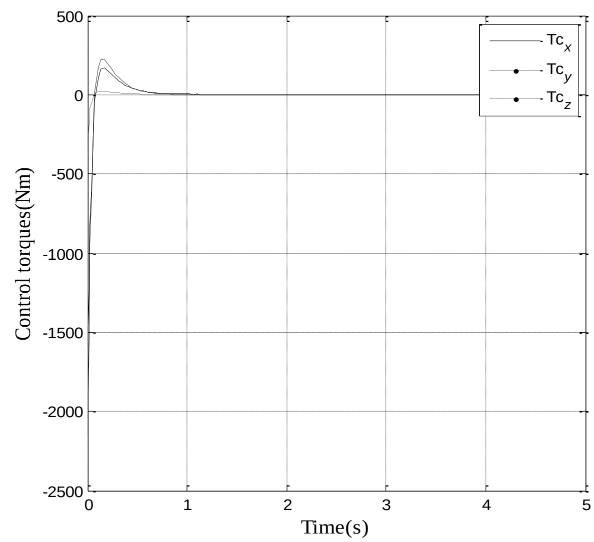

FIG. 4 Control torques

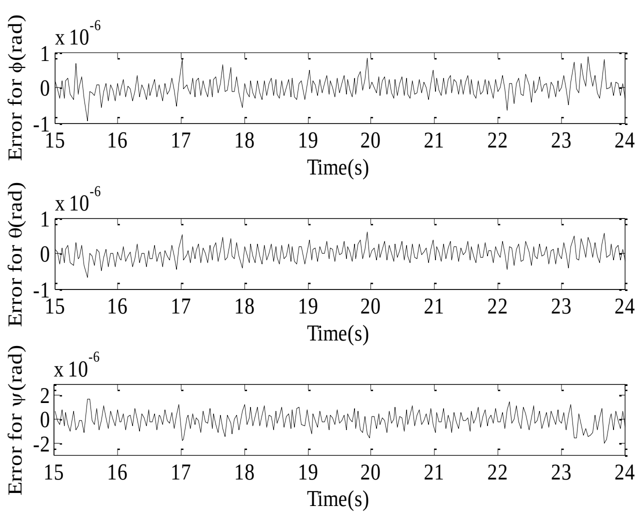

FIG. 5 Error for attitude angle

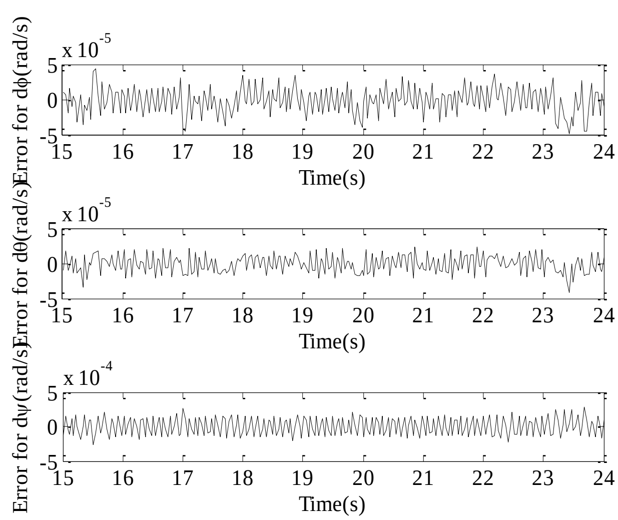

FIG. 6 Error for attitude angle changing rate

FIG. 1-FIG. 6 show the simulation results with the non-fragile H_∞ controller under the condition of additive perturbation with **Theorem1**. FIG. 1 and FIG. 2 show the measurable output of attitude angles and angular velocity, respectively, from which we can see that the attitude control system can

achieve a steady state within approximately 3s with quick response. FIG. 3 shows the external disturbance torques of white noise. When simulation is conducted, the amplitude of the noise is assumed bigger than actual value to show the strength of the proposed method. FIG. 4 shows the control torques of the actuator without considering the maximal control input. FIG. 5 and FIG. 6 show the error for attitude angle and its changing rate when the satellite attitude system is of a steady state chosen between 15s and 24s here. The absolute errors are all at 10-4 or even 10-6 orders of magnitude which are so small as to satisfy the required accuracy.

To demonstrate the good performance of the proposed method in this paper, a mixed H_2/H_∞ control method in [10] is used for simulation, the results can be seen as follows.

With **Theorem2** we can attain the corresponding state feedback gain as follow

$$K=\begin{bmatrix} -0.0330 & 0 & 6.0048e\text{-}14 & -4.7079 & 0 & -0.0347 \\ 0 & -0.0333 & 0 & 0 & -4.7079 & 0 \\ 2.3068e\text{-}13 & 0 & -0.0862 & 0.0347 & 0 & -2.9019 \end{bmatrix}$$

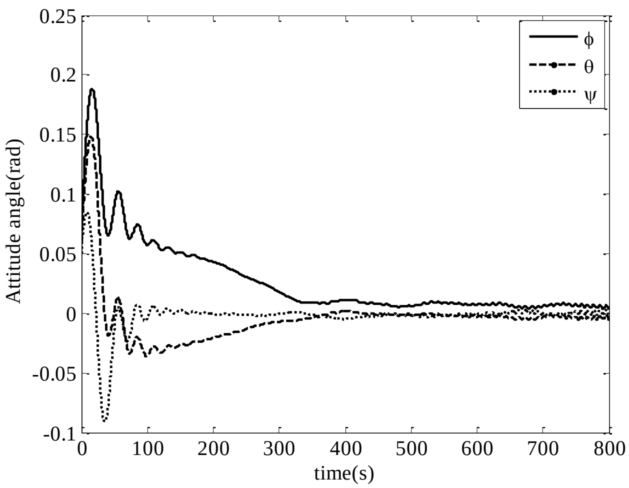

FIG.7 Attitude angle with mixed method

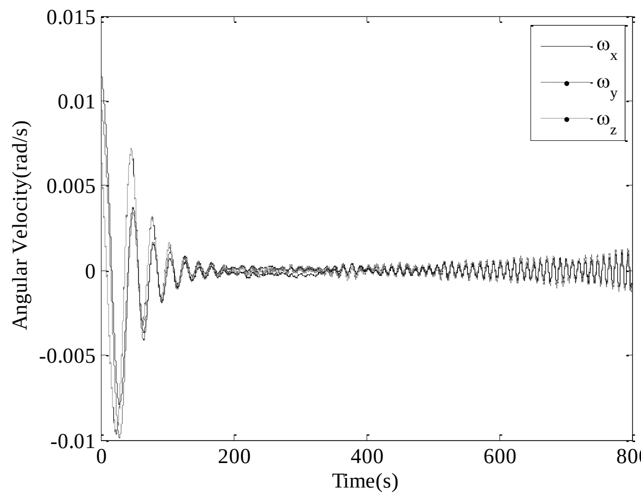

FIG.8 Angular velocity with mixed method

The attitude angle and angular velocity with mixed H_2/H_∞ method can be seen in FIG.7 and FIG.8. Obviously, with uncertainties and other disturbances, the satellite attitude control system can have a better robustness with the method in this paper. Furthermore, the system can response much more quickly can have much less error with the robust non-fragile H_∞ control method.

5. Conclusion

This paper proposes a new robust non-fragile H_∞ controller design method for satellite attitude control system with parameter uncertainties and external disturbances. The design of the controller is subject to the constraints of H_∞ performance and quadratic stability. By using LMI techniques resulting from Lyapunov theory, the satellite attitude control problem is transformed into a convex optimization problem with LMI constraints. Compared with the results of mixed H_2/H_∞ method, the proposed method on uncertain satellite attitude control system has better practicability and effectiveness. Further research in terms of multiplicative perturbation is under way.

References

[1] Lien C H. Non-fragile guaranteed cost control for uncertain neutral dynamic systems with time-varying delays in state and control input. Chaos, Solitons & Fractals, 2007, 31(4): 889-899.

[2] Zhang J, Shi P, Qiu J. Non-fragile guaranteed cost control for uncertain stochastic nonlinear time-delay systems. Journal of the Franklin Institute, 2009, 346(7): 676-690.
[3] Lee D H, Park J B, Joo Y H, et al. Robust H∞ control for uncertain nonlinear active magnetic bearing systems via Takagi-Sugeno fuzzy models. International Journal of Control, Automation and Systems, 2010, 8(3): 636-646.
[4] Hou Y Y, Liao T L, Yan J J, et al. Non-fragile H∞ control for singular systems with state and input time-varying delays. International Journal of Nonlinear Sciences and Numerical Simulation, 2007, 8(1): 31-40.
[5]Yang J, Zhong S, Xiong L. A descriptor system approach to non-fragile H∞ control for uncertain fuzzy neutral systems. Fuzzy sets and systems, 2009, 160(4): 423-438.
[6] Keel L H, Bhattacharyya S P. Robust, fragile, or optimal. Automatic Control, IEEE Transactions on, 1997, 42(8): 1098-1105.
[7]Yonchev A, Gilev B, Penev D. Sensitivity Analysis of the Continuous-time LMI-based H∞ Quadratic Stability Problem. American Institute of Physics Conference Series. 2010, 1293: 3-12.
[8] Yang G H, Wang J L. Nonfragile H∞ Output Feedback Controller Design for Linear Systems. Journal of dynamic systems, measurement, and control, 2003, 125(1): 117-123.
[9] Fang M, Park J H. Non-fragile synchronization of neural networks with time-varying delay and randomly occurring controller gain fluctuation. Applied Mathematics and Computation, 2013, 219(15): 8009-8017.
[10] Chuang Liu, Feng Wang, Zongzeng Zhang. Mixed H_2/H_∞ control for a satellite based on LMI. International Conference on Mechatronics and Control, Jinzhou, 3-5 July 2014.
[11] Gao C, Zhao Q, Duan G. Robust actuator fault diagnosis scheme for satellite attitude control systems. Journal of the Franklin Institute, 2013, 350(9): 2560-2580.