Design and Simulation of Fractional Order PID Controllers Based on Bode's Ideal Transfer Function

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Abstract. The fractional order calculus theory and its modeling methods have been applied widely in control field. And the design of the fractional controller becomes the hot point of recent years. This paper establishes fractional order controllers which parameters are obtained by Bode's ideal transfer function method to get the desired frequency response. Controllers are designed for integral first order system and for fractional first order system, the simulation results indicate that the effectiveness and validity of this method.

Keywords: Fractional order PID; Bode's ideal transfer function; Al-Alaoui+CFE; simulation; robustness.

1. Introduction

Fractional calculus deals with derivatives and integrals to an arbitrary order.There are several definitons of fractional derivatives and integrals, the most fundamental definition of a fractional derivative and integral of order α is given by Grünwald-Letnikov definition [1, 3, 5], Grünwald-Letnikov (GL) definition is given as:

$$ {}_aD_t^{\alpha} f(t) = \lim_{h\to 0} h^{-\alpha} \sum_{j=0}^{[\frac{t-a}{h}]} (-1)^j \binom{\alpha}{j} f(t-jh) \tag{1}$$

Where, $\binom{\alpha}{j} = \frac{\alpha(\alpha+1)...(\alpha+j-1)}{j!} = \frac{\alpha!}{j!(\alpha-j)!}$.

When $\alpha > 0$, means α is derivative order of $f(t)$;when $\alpha < 0$,mean α is integral order of $f(t)$. This definition is wildly used in control field[5].

Design the fractional controller is a main researching area of fractional calculus applying in the control field. And there are many kinds of methods to design a fractional order PID (FOPID) controller, including the phase margin, the amplitude margin, dominant pole method, optimization method, and the Bode's ideal transfer function method. The Bode's ideal transfer function method is easy and effective .It suggested an ideal shape of the open-looptransfer function of the form[5]:

$$ G_{opi}(s) = \left(\frac{\omega_c}{s}\right)^{\alpha}, \alpha \in R \tag{2}$$

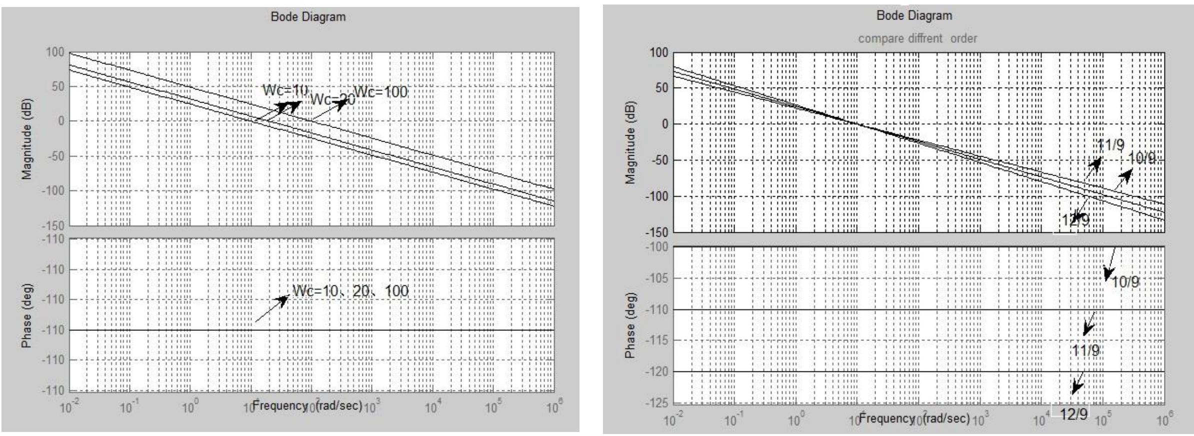
Where ωc is the gain crossover frequency, in fact, thetransfer function $G_{opi}(s)$ is a fractional-order differentiator for $\alpha > 0$ and a fractional-order integrator for $\alpha < 0$.

Its Open-loop characteristics are as follows:

Amplitude-frequency curve is is a straight line of constant slope−20α dB/dec;

Phase curve is a horizontal line at−α π/2 rad;

The Nyquist curve consists, simply, on a straight line through the origin with argGopi(jω)=− α π/2 rad.



a)Frequency and phase curves when fix α =11/9 and changing ωc

b) Frequency and phase curves when fix ωc =10rad/s and changing α

Fig.1 Frequency and phase curves When parameter changing

Fig.1 indicate that if the gain changes ωc, will changes together but the phase margin of the system remains as a independent value as $\phi_m = (\pi - 2\alpha/\pi)$ rad[5].In this paper we applied the Bode's ideal transfer function method to the determine the parameters of the FOPID controllers for integral first order system and fractitional first order system. The prosedures of design FOPID controllers using Bode's ideal transfer function method are as follows:

Choosen the controlled plant and its transfer function $G_P(S)$;

Assume gain crossover frequency ωc, the phase marginΦm; Work out the transfer function of FOPID controller

$$G_c(s) = \frac{G_{opi}(s)}{G_P(S)} \tag{3}$$

Using impulse invariance method get the discretization model of $G_c(s)$ and the simulation block of FOPID controller, Schematic Diagram is as Fig.2 [13].

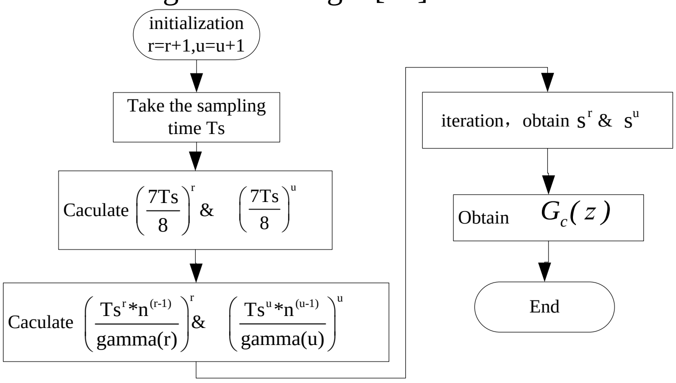


Fig.2 Schematic diagram of the controller discretization process

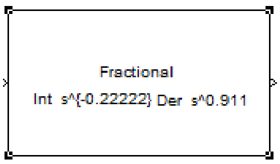


Fig.3 The simulation block of fractional controller

2. Establishing FOPID Controller Model

2.1 Establishing FOPID Controller Model Aimed at Integer First-Order Control Plant

Assume the open-loop transfer function of control object is:

$$G_{P1}(s) = \frac{1}{Ts+1} \quad (4)$$

here, T=0.4s, hypothesis Φm=70o, ωc =10rad/s.

As prosedures(2) ,the transfer function of controller aimed at integer first-order control plant based on Bode's ideal transfer function can be get from formula (3) :

$$G_{c1}(s) = \frac{G_{opi}(s)}{G_P(S)} = \left(10^{\frac{11}{9}}\right)\left(0.4s^{-\frac{2}{9}} + s^{-\frac{11}{9}}\right)$$

The simulation model of FOID controller aimed at integer first-order control plant is shown in Fig.4.

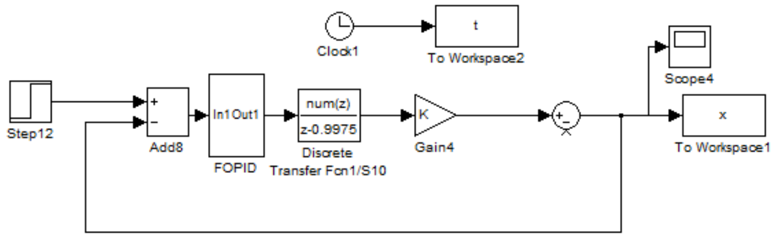


Fig. 4 Simulation model of FOPID controller aimed at integer first-order control plant

2.2 Establishing FOPID Controller Model Aimed at Fractional First-Order Control Plant

Assume the open-loop transfer function of control object is:

$$G_{P2}(s) = \frac{1}{Ts^{\gamma}+1} \quad (5)$$

here, T=0.5s, $\gamma = 0.5$ hypothesis Φm=70o, ωc =10rad/s.

As prosedures (2), the transfer function of controller aimed at fractional first-order control plant based on Bode's ideal transfer function can also be get from formula (3) :

$$G_{c2}(s) = \frac{G_{opi}(s)}{G_P(S)} = \left(10^{\frac{11}{9}}\right)\left(0.5s^{-\frac{13}{18}} + s^{-\frac{11}{9}}\right) \quad (6)$$

The simulation model of FOID controller aimed at integer first-order control plant is shown in Fig.5.While the subsysytem FOPID and FOSystem are designed of the simulation blocks shown in Fig.3 applying the parameters in equation(5), (6).

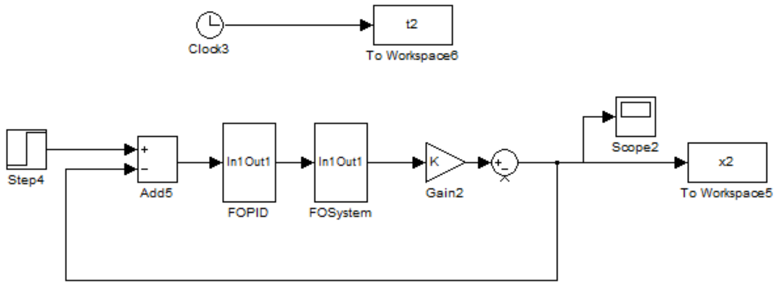
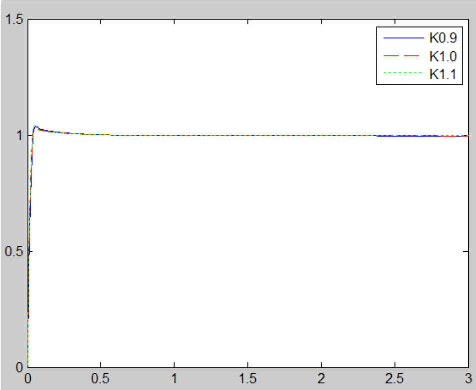
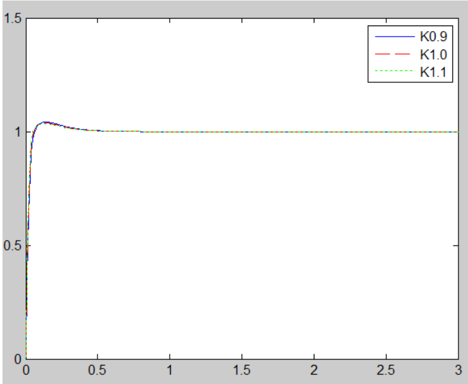


Fig. 4 Simulation model of FOPID controller aimed at fractional first-order control plant

3. Simulation and Result Analysis

Unit Step Responses with open-loop system gain K varying (0.9K, K, 1.1 K) are shown in Fig.5. 6. Fig.5 shows that the controllers designed based on Bode's ideal transfer function have both robustnss anti-to the variation of system open-loop gain.





a) Unit step responses of integer first-order control Plant b) Unit step responses of fractional r first-order control plant

Fig. 5 Unit step responses

4. Conclusion

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With the Bode's ideal transfer function method, we obtain closed-loop systems robust to gain variations and the step responses indicating an iso-damping property. And the caculating process is fairely easy to traditional fractional order controller design methods. But it must be noted that the characteristic has limitation to the gain crossover frequency ωc and that the phase margin of the resulting closed-loop system is not exactly identical to the prescribed value defined by the slope α at that frequency. This is due to realize a arbitrary order by physical facilities are ususlly not easily. simulations illustrate that the order α can be confirmed easily, but the select of ωc must consider a lot. To obtain a good performace, ωc must be as high as possible. And if too high this will hardly to get the PID parameters.

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