A Study on Control Strategy for Regenerative Braking in Parallel Hybrid Electric Vehicles Based on Energy Recovery Sufficiency

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Abstract. The introduction of electric motors brings in the function of regenerative braking for vehicles. In order to increase the regenerative braking recovery rate, a parallel braking force distribution strategy for passenger cars with front-wheel drive pattern was discussed. The traditional parallel strategy was introduced at first. Based on its defects, the revised parallel strategy was proposed. The advantages of the revised one were analyzed through theoretical derivation from recovery rate to the braking distance. The braking models for the two mentioned strategies were built and simulated on the platform of MATLAB/SIMULINK. The results indicate that the revised strategy can recover more energy and shorten the braking distance at the same time.

Keywords: Parallel hybrid electric vehicles, Parallel braking force distribution strategy, Regenerative braking, Recovery rate, Braking distance

1. Introduction

Hybrid cars, transitional products between internal-combustion engine vehicles (ICEVs) and pure electric vehicles, would be the main products in automobile works [1-2]. This is because they alleviate the serious crisis of oil shortage and environmental pollution caused by ICEVs and because they address the storage problems of cells in electric cars at the same time [3]. The electric motors help the hybrid cars achieve the goals of regenerative braking [4]. Thus the problem of recovering more energy occurs [5-6].

2. Traditional parallel strategy

The regenerative braking forces are added on front wheels.

$$F_{b_{f_{sum}}} = F_{b_{f_{me}}} + F_{b_{f_{re}}}$$
(1)

$$F_{b_r_sum} = F_{b_r_me} \tag{2}$$

As is shown in Eq.1 and Eq.2: $F_{b_{f_sum}}$ are the sum of braking forces on front wheels; $F_{b_{r_sum}}$ are the sum of braking forces on rear wheels; $F_{b_{f_re}}$ are the regenerative braking forces on front wheels; $F_{b_{f_re}}$ are the mechanical braking forces on front wheels; $F_{b_{f_re}}$ are the mechanical braking forces on front wheels; $F_{b_{r_sme}}$ are the mechanical braking forces on front wheels; $F_{b_{r_sme}}$ are the mechanical braking forces on front wheels; $F_{b_{r_sme}}$ are the mechanical braking forces on front wheels; $F_{b_{r_sme}}$ are the mechanical braking forces on front wheels; $F_{b_{r_sme}}$ are the mechanical braking forces on front wheels; $F_{b_{r_sme}}$ are the mechanical braking forces on front wheels; $F_{b_{r_sme}}$ are the mechanical braking forces on front wheels; $F_{b_{r_sme}}$ are the mechanical braking forces on front wheels; $F_{b_{r_sme}}$ are the mechanical braking forces on front wheels; $F_{b_{r_sme}}$ are the mechanical braking forces on front wheels; $F_{b_{r_sme}}$ are the mechanical braking forces on front wheels; $F_{b_{r_sme}}$ are the mechanical braking forces on front wheels; $F_{b_{r_sme}}$ are the mechanical braking forces on front wheels; $F_{b_{r_sme}}$ are the mechanical braking forces on front wheels; $F_{b_{r_sme}}$ are the mechanical braking forces on front wheels; $F_{b_{r_sme}}$ are the mechanical braking forces on front wheels; $F_{b_{r_sme}}$ are the mechanical braking forces on front wheels; $F_{b_{r_sme}}$ are the mechanical braking forces on front wheels; $F_{b_{r_sme}}$ are the mechanical braking forces on front wheels; $F_{b_{r_sme}}$ are the mechanical braking forces on front wheels; $F_{b_{r_sme}}$ are the mechanical braking forces on front wheels; $F_{b_{r_sme}}$ are the mechanical braking forces on front wheels; $F_{b_{r_sme}}$ are the mechanical braking forces on front wheels; $F_{b_{r_sme}}$ are the mechanical braking forces on front wheels; $F_{b_{r_sme}}$ are the mechanical braking forces on front wheels; $F_{b_{r_sme}}$

The braking force distribution coefficient of PHEV with front-wheel drive pattern is the ratio of the sum of front mechanical braking force and front regenerative braking force to the sum of mechanical braking force and regenerative braking force.

$$\beta_{hev} = \frac{F_{b_{f_me}} + F_{b_{f_re}}}{F_{b_{f_me}} + F_{b_{f_re}} + F_{b_{f_re}}}$$
(3)

As is shown in Eq.3, β_{hev} is the braking force distribution coefficient of PHEV with front-wheel pattern.

The traditional parallel strategy divides the brake process into three parts. When the expected braking severity is lower than 0.1, the PHEV is in mild braking period. When the expected braking

severity is lower than 0.7 but higher than 0.1 and the mechanical braking forces have not made the front or rear wheels being in critical locked state, the PHEV is in moderate braking period. When the braking severity is higher than 0.7 or the mechanical braking forces have made the front or rear wheels being in critical locked state, the PHEV is in severe braking period.

In the mild braking period, the regenerative braking forces are added on front wheels and the mechanical braking forces of front and rear wheels are zero.

$$F_{b_f_re} = F_{re_a} \tag{4}$$

$$F_{b_{-}f_{-}me} = F_{b_{-}r_{-}me} = 0 \tag{5}$$

As is shown in Eq.4, F_{re_a} are the maximum regenerative braking forces that are provided by the motor.

In the moderate braking period, both the regenerative braking forces and the mechanical braking forces contribute to the braking operation. The value of regenerative braking force is decided by the ECE braking regulations and the constraint of anti-lock front wheels.

The limitations of ECE braking regulations reflect in the braking force distribution coefficient.

$$\beta_{hev_{max}} = (b + zh_g)(z + 0.07) / (0.85Lz)$$
(6)

As is shown in Eq.6, β_{hev_max} is the maximum braking force distribution coefficient allowed by ECE; *b* is the distance between center of mass and rear axle; h_g is the height of center of mass; *L* is the wheel base of vehicle; *z* is the severity of braking.

$$F_{b_{-f_{-}re}} \le \frac{(\beta_{hev_{-}\max} - 1)F_{b_{-f_{-}me}} + \beta_{hev_{-}\max}F_{b_{-}r_{-}me}}{1 - \beta_{hev_{-}\max}} = F_{b_{-}f_{-}re_{-}ECE}$$
(7)

As is shown in Eq.7, $F_{b_{f_re_ECE}}$ is the maximum regenerative braking force on front wheels allowed by ECE.

The constraints of anti-lock front wheels can be concluded by the f curve.

$$F_{b_{f_{lock}}} = \frac{F_{b_{r_{sum}}} + Gb / h_g}{\left(L - \varphi h_g\right) / \varphi h_g}$$
(8)

$$F_{b_f_re} \le F_{b_f_lock} - F_{b_f_me} = F_{b_f_re_ABS}$$

$$\tag{9}$$

As is shown in Eq.8 and Eq.9, $F_{b_{-}f_{-}lock}$ is the maximum braking force on front wheels allowed by anti-lock front wheels constraint; φ is the pavement friction coefficient; *G* is the vehicle gravity; $F_{b_{-}f_{-}re_{-}ABS}$ is the maximum regenerative braking force on front wheel allowed by anti-lock front wheels constraint.

Eq. 10 shows the maximum regenerative braking force in the moderate braking period.

$$F_{b_f_re_max} = \min\left\{F_{b_f_re_ECE}, F_{b_f_re_ABS}\right\}$$
(10)

Eq.11 shows the participant regenerative braking force in the moderate braking period.

$$F_{b_{-}f_{-}re} = \begin{cases} F_{re_{-}a}, & F_{re_{-}a} \leq F_{b_{-}f_{-}re_{-}\max} \\ F_{b_{-}f_{-}re_{-}\max}, & F_{re_{-}a} > F_{b_{-}f_{-}re_{-}\max} \end{cases}$$
(11)

In the severe braking period, only the mechanical braking forces play roles in braking process; the regenerative braking force is zero.

$$F_{b_f_re} = 0 \tag{12}$$

During the whole braking process, the ratio of front mechanical braking force to rear mechanical braking force is a constant value.

$$\beta_{me} = \frac{F_{b_{f_me}}}{F_{b_{f_me}} + F_{b_{r_me}}}$$
(13)

As is shown in the Eq.13, β_{me} is the mechanical braking force distribution coefficient.

It can be concluded from Eq.13 that if the regenerative braking force participate in the braking process, the braking force distribution coefficient of PHEV with front-wheel drive pattern would be higher than the mechanical braking force distribution coefficient.

The mechanical braking force distribution is determined by the vehicle parameters and the synchronizing adhesion coefficient.

$$\varphi_0 = \left(L\beta_{me} - b \right) / h_g \tag{14}$$

As is shown in the Eq.14, φ_0 is the synchronizing adhesion coefficient.

In sum, the mechanical braking force is decided by the driver intention. In the mild braking period, only the regenerative braking forces play roles in braking process. In the moderate braking period, both the regenerative braking forces and the mechanical braking forces play roles in braking process. In the severe braking period, only the mechanical braking forces play roles in braking process.

3. Revised parallel strategy

In the traditional parallel strategy, the PHEV can recover lots of energy in the mild braking period; it can recover considerable energy and shorten the braking distance in the moderate braking period; however it can recover little energy in the severe braking period.

If the PHEV is running on the road whose friction coefficient is over 0.7, regenerative braking force on front wheels can still be added when the PHEV is just at the beginning of the severe braking period because the mechanical braking forces have not made the front (rear) wheels being in critical locked state. If the PHEV is running on the road whose friction coefficient is higher than the synchronizing adhesion coefficient, the mechanical braking forces make the rear wheel being in the critical locked state in the severe braking period. At this time, the regenerative braking force can still be added on the front wheels until the front wheels slide into the critical locked state.

In order to increase the regenerative braking recovery rate, the novel strategy would revise in the two mentioned cases.

Eq.15 shows the revised strategy in the mild braking period. In this period, the revised one is the same with the traditional one.

$$\begin{cases}
F_{b_{-}f_{-}re} = F_{re_{-}a} \\
F_{b_{-}f_{-}me} = F_{b_{-}r_{-}me} = 0 \\
F_{b_{-}f_{-}sum} = F_{b_{-}f_{-}re} \\
F_{b_{-}r_{-}sum} = 0
\end{cases}$$
(15)

When the PHEV finishes the mild braking period and the mechanical braking forces have not made the front (rear) wheels being in critical locked state, the regenerative braking forces are decided by the ECE braking regulations and the constraint of anti-lock front wheels. Eq.16 shows the strategy in this case.

$$\begin{cases}
F_{b_{-f_{-}re_{-}\max}} = \min \left\{ F_{b_{-f_{-}re_{-}ECE}}, F_{b_{-f_{-}re_{-}ABS}} \right\} \\
F_{b_{-f_{-}re}} = \left\{ F_{re_{-}a}, & F_{re_{-}a} \le F_{b_{-}f_{-}re_{-}\max} \\
F_{b_{-}f_{-}re_{-}\max}, & F_{re_{-}a} > F_{b_{-}f_{-}re_{-}\max} \\
F_{b_{-}f_{-}sum} = F_{b_{-}f_{-}me} + F_{b_{-}f_{-}re} \\
F_{b_{-}r_{-}sum} = F_{b_{-}r_{-}me}
\end{cases}$$
(16)

When the PHEV finishes the mild braking period and the mechanical braking forces have made the front (rear) wheels being in critical locked state, the regenerative braking forces are decided by the constraint of anti-lock front wheels. Eq.17 shows the strategy in this case.

$$\begin{cases} F_{b_{-f_{-}re_{-}\max}} = F_{b_{-f_{-}re_{-}ABS}} \\ F_{b_{-f_{-}re}} = \begin{cases} F_{re_{-}a}, & F_{re_{-}a} \leq F_{b_{-}f_{-}re_{-}\max} \\ F_{b_{-}f_{-}re_{-}\max}, & F_{re_{-}a} > F_{b_{-}f_{-}re_{-}\max} \\ F_{b_{-}f_{-}sum} = F_{b_{-}f_{-}me} + F_{b_{-}f_{-}re} \\ F_{b_{-}f_{-}sum} = F_{b_{-}f_{-}me} \end{cases}$$
(17)

In a nutshell, the revised strategy can recover certain energy in some cases in the severe braking strategy.

4. Comparison of recovery rate

The recovery rate of the two strategies are compared when both the PHEV with different strategies are running on the same road with the same drive intention and initial velocities.

When the traditional PHEV is in the mild and moderate braking periods, the revised PHEV has the same states with the traditional PHEV. So the recovered energy of the two is equal in these two periods.

$$Energy_{hev t} - partA = Energy_{hev t} - partA$$
(18)

As is shown in the Eq.18, $Energy_{hev_t} - partA$ is the traditional PHEV recovered energy in the mild and moderate braking period; $Energy_{hev_r} - partA$ is the revised PHEV recovered energy in the mild and moderate braking period.

When the synchronizing adhesion coefficient is higher than the pavement friction coefficient, the pavement friction coefficient is lower than 0.7, and the traditional PHEV is in severe braking period, the revised PHEV has the same strategy with the traditional PHEV. So the recovered energy of the two is equal at this time.

$$Energy_{hev t} - partB = Energy_{hev t} - partB = 0$$
(19)

As is shown in the Eq.19, $Energy_{hev_t} - partB$ is the traditional PHEV recovered energy in the severe braking period; $Energy_{hev_r} - partB$ is the revised PHEV recovered energy in the severe braking period.

When the synchronizing adhesion coefficient is lower than the pavement friction coefficient and the traditional PHEV is in severe braking period, the revised PHEV adds more regenerative braking forces than the traditional PHEV does. Eq.20 shows the recovered energy relation of the two PHEV at this time.

$$Energy_{hev r} _ partB > Energy_{hev t} _ partB = 0$$
⁽²⁰⁾

When the pavement friction coefficient is higher than 0.7 and the traditional PHEV is in severe braking period, the revised PHEV adds more regenerative braking forces than the traditional PHEV does. Eq.21 shows the recovered energy relation of the two PHEV at this time.

$$Energy_{hev} = partB > Energy_{hev} = partB = 0$$
(21)

Eq.22 shows the recovered energy of the traditional PHEV during the whole braking process.

$$Energy_{hev t} = Energy_{hev t} _ partA + Energy_{hev t} _ partB$$
(22)

As is shown in Eq.22, $Energy_{hev_t}$ is the recovered energy of the traditional PHEV during the whole brake process.

Eq.23 shows the recovered energy of the revised PHEV during the whole braking process.

$$Energy_{hev_r} = Energy_{hev_r} - partA + Energy_{hev_r} - partB$$
(23)

As is shown in Eq.23, $Energy_{hev_r}$ is the recovered energy of the revised PHEV during the whole brake process.

Eq.24 shows the relation of the recovered energy.

$$Energy_{hev_t} \le Energy_{hev_r}$$
 (24)

Eq.25 shows the relation of the recovery rate.

$$0 < \frac{Energy_{hev_{-}t}}{\frac{1}{2}Mv_0^2} \le \frac{Energy_{hev_{-}r}}{\frac{1}{2}Mv_0^2} < 1$$
(25)

As is shown in Eq.25, M is the vehicle mass; v_0 is the initial velocity.

It can be concluded from the Eq.25 that the revised strategy can recover more energy than the traditional one.

5. Comparison of braking distance

The braking distance of the two strategies are compared when both the PHEV with different strategies are running on the same road with the same drive intention and initial velocities.

When the traditional PHEV is in the mild and moderate braking periods, the revised PHEV has the same states with the traditional PHEV. So the braking distance of the two is equal in these two periods.

$$Distance_{hev t} = partA = Distance_{hev t} = partA$$
(26)

As is shown in the Eq.26, $Distance_{hev_t} _ partA$ is the traditional PHEV braking distance in the mild and moderate braking period; $Distance_{hev_r} _ partA$ is the revised PHEV braking distance in the mild and moderate braking period.

When the synchronizing adhesion coefficient is higher than the pavement friction coefficient, the pavement friction coefficient is lower than 0.7, and the traditional PHEV is in severe braking period, the revised PHEV has the same strategy with the traditional PHEV. So the braking distance of the two is equal at this time.

$$Distance_{hev,t} _ partB = Distance_{hev,r} _ partB$$
(27)

As is shown in the Eq.27, $Distance_{hev_t} _ partB$ is the traditional PHEV braking distance in the severe braking period; $Distance_{hev_r} _ partB$ is the revised PHEV braking distance in the severe braking period.

When the synchronizing adhesion coefficient is lower than the pavement friction coefficient and the traditional PHEV is in severe braking period, the revised PHEV adds more regenerative braking forces than the traditional PHEV does. The revised PHEV has higher braking severity than the traditional PHEV does. Eq.28 shows the braking distance relation of the two PHEV at this time.

$$Distance_{hev_t} _ partB > Distance_{hev_t} _ partB$$
(28)

When the pavement friction coefficient is higher than 0.7 and the traditional PHEV is in severe braking period, the revised PHEV adds more regenerative braking forces than the traditional PHEV does. The revised PHEV has higher braking severity than the traditional PHEV does. Eq.29 shows the braking distance relation of the two PHEV at this time.

$$Distance_{hev,t} _ partB > Distance_{hev,t} _ partB$$
 (29)

Eq.30 shows the braking distance of the traditional PHEV during the whole braking process.

$$Distance_{hev,t} = Distance_{hev,t} - partA + Distance_{hev,t} - partB$$
(30)

As is shown in Eq.30, $Distance_{hev_t}$ is the braking distance of the traditional PHEV during the whole brake process.

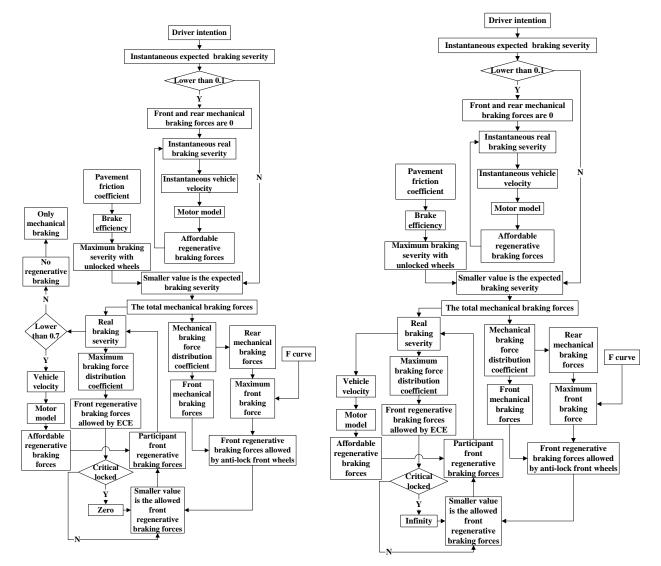
Eq.31 shows the braking distance of the revised PHEV during the whole braking process.

$$Distance_{hev_r} = Distance_{hev_r} - partA + Distance_{hev_r} - partB$$
(31)

As is shown in Eq.31, $Distance_{hev_r}$ is the braking distance of the revised PHEV during the whole brake process.

Eq.32 shows the relation of the braking distance.

$$Distance_{hev t} \ge Distance_{hev t}$$
(32)



It can be concluded from the Eq.32 that the revised strategy can shorten the braking distance.

Fig.1 Flow chart of the traditional strategy model

Fig.2 Flow chart of the revised strategy model

6. Braking models

The traditional strategy and the revised one were built on the platform of MATLAB/SIMULINK. Fig.1 shows the flow chart of the traditional strategy model; Fig.2 shows the flow chart of the revised strategy model.

The model in the Fig.1 has a few differences compared with the model in the Fig.2. The model in the Fig.2 cancels the comparison between 0.7 and the braking severity. And the model in the Fig.2 uses the value of infinity instead of zero in the ECE constraint.

7. Analyses of simulation results

In order to compare the braking distance of the two strategies, the driver intention adopts the process of slowly stepping on the brake pedal instead of the NEDC driving cycle. The process of slowly stepping on the brake pedal contains the mild braking period, the moderate braking period and the severe braking period.

The two models, embed with the parameters shown in Table 1, were simulated. The simulation results are shown from Fig.3 to Fig.7. In detail: Fig.3 shows the front regenerative braking forces; Fig.4 shows the braking severity; Fig.5 shows the braking force distribution coefficient; Fig.6 shows the recovered energy; Fig.7 shows the braking distance.

Fig.3 shows that the front regenerative braking force of the traditional strategy had some vibrations during some period. This is because the PHEV was in the transition state between the moderate braking period and the severe braking period at that time. After the vibrations, the traditional strategy no longer provided the regenerative braking force. However, the revised strategy provided the regenerative braking process.

Table 1 Parameters

Parameters	Value
Wheel base of PHEV [m]	2.6
Distance between front axle and the mass center [m]	1.04
Distance between rear axle and the mass center [m]	1.56
Height of center of mass [m]	0.5
Mass of PHEV [kg]	1600
Wheel radius [m]	0.25
Power of DC motor [kW]	30
Pavement friction coefficient	0.8
Synchronizing adhesion coefficient	0.7
Mechanical braking force distribution coefficient	0.7346
Initial velocity [km/h]	50

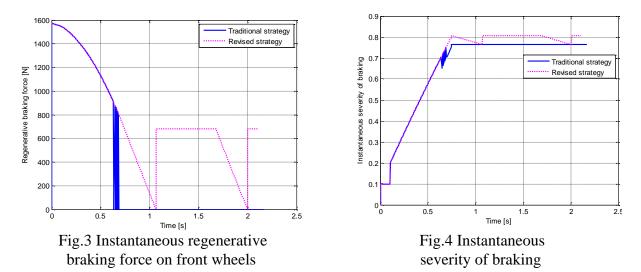


Fig.4 shows that the instantaneous severity of braking of the traditional strategy had some vibrations during some period. This is because the front regenerative braking force of the traditional strategy had some vibrations at that time. The maximum of the braking severity of the traditional strategy was lower than the pavement friction coefficient. However, the maximum of the braking severity of the revised strategy equaled the pavement friction coefficient. This is because the front regenerative braking force made the front wheels being in the critical locked state in the severe period.

Fig.5 shows that the instantaneous braking force distribution coefficient of the traditional strategy had some vibrations during some period. This is because the front regenerative braking force of the traditional strategy had some vibrations at that time. The minimum of the braking force distribution coefficient of the traditional strategy equaled the mechanical braking force distribution coefficient. However, the braking force distribution coefficient of the revised strategy was not lower than the mechanical braking force distribution coefficient during the whole braking process. This is because the front regenerative braking forces were added during the whole period.

Fig.6 shows that the traditional strategy did not recover the energy in the severe braking period. However, the revised strategy was recovering the energy during the whole braking process.

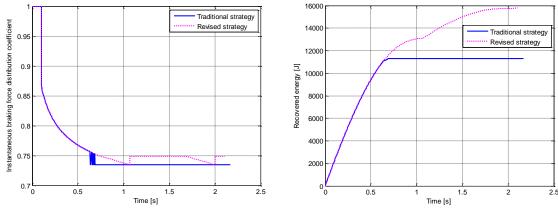


Fig.5 Instantaneous braking force distribution coefficient

Fig.6 Recovered energy

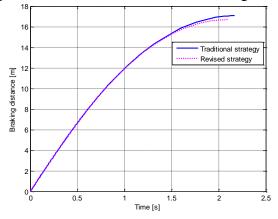


Fig.7 Braking distance Table 2 Simulation results

Pavement friction coefficient	Traditional strategy recovery rate	Revised strategy recovery rate	Increment rate of recovery rate	Traditional strategy braking distance	Revised strategy braking distance	Decrement rate of braking distance
0.5	5.15%	5.15%	0%	23.21	23.21	0%
0.6	6.27%	6.27%	0%	19.90	19.90	0%
0.7	7.23%	7.23%	0%	17.79	17.79	0%
0.8	7.33%	10.22%	39.43%	17.07	16.70	2.17%
0.9	7.33%	11.28%	53.89%	16.57	16.08	2.96%

Fig.7 shows that the braking distance of traditional strategy is the same with the counterpoint of revised strategy in the mild and moderate braking period. In addition, the traditional strategy had longer braking distance than the revised strategy in the severe braking period.

Additionally, the two models, embed with the parameters shown in Table 1, were simulated with different pavement friction coefficients. Table 2 shows the results.

The results in the Table 2 indicate that the revised strategy is able to recover energy in the severe braking period in some cases. When the pavement friction coefficient is 0.8, there is an astounding 39.43% increase in recovery rate and an astounding 2.17% decrease in braking distance. When the pavement friction coefficient is 0.9, there is an astounding 53.89% increase in recovery rate and an astounding 2.96% decrease in braking distance.

8. Conclusion

The object of study is parallel hybrid electric passenger cars with front-wheel drive pattern. The traditional strategy was revised and the novel strategy can recover energy in the severe braking period in some cases. In these cases, the PHEV with revised strategy can not only increase the recovery rate but also shorten the braking distance.

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