Reliability analysis of cantilever beam structure based on multiplier method and immune algorithm

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Abstract

When the traditional reliability optimization design method deals with the practical reliability engineering problems, it results in the situation that the accuracy of solution is not high. Based on this, this paper proposes a hybrid reliability analysis method based on augmented multiplier and immune algorithm. Firstly, the mathematical model of reliability optimization design is established taking the limit state equation as the constraint and the reliability index as the objective function. Secondly, the mathematical model of constrained reliability optimization design is transformed into the unconstrained optimization model by the augmented multiplier method. Finally, the reliability index is solved by using the immune algorithm (IA). The effectiveness and nonlinear solving ability of the proposed method are verified by numerical examples; the application ability of the proposed method to practical engineering problems is verified by engineering examples, and the influence of correlation on reliability index is discussed in engineering examples.

Keywords

Reliability; Augmented multiplier method; Immune algorithm; Correlation.

1. Introduction

With the increasing complexity and refinement requirements of modern machine structure, it is urgent to evaluate the reliability optimization of the project by a calculation model which considers many factors comprehensively. Therefore, in order to solve the practical engineering problems, many researchers put forward lots of new calculation methods [1-5]. Zhang [6] used ahp-rbf neural network to evaluate the reliability of the mine ventilation system. Guo [7] used FOSM to analyze the motion reliability of the mechanism. Han [8] analyzed the reliability of the gyratory crusher by using the method of fault tree analysis. Liu [9] established the performance degradation model from the point of failure physical analysis, and integrated the life model and performance degradation model to obtain the reliability evaluation model by Bayesian method.

The existing reliability evaluation methods have some limitations on the problem and do not take the parameter correlation into account. For example, the first order second moment (FOSM) cannot be applied to the functions that are difficult to derive or have strong nonlinearity. In this paper, a hybrid reliability analysis method based on Augmented Lagrangian multiplier and immune algorithm is proposed. The mathematical model is established by taking the limit state equation as the constraint and the reliability index as the objective function. The constrained problem is transformed into the unconstrained problem by the augmented multiplier method. At the same time, the influence of the initial penalty factor on the solution result is avoided, and the parameter correlation is considered. Immune algorithm is used to solve the model. Numerical experiments show that the immune algorithm has good convergence characteristics for nonlinear and linear problems, and its reliability index is closer to that of Monte Carlo simulation (MCS).

2. The establishment of mathematical model for reliability optimization design

2.1. Establishment of reliability index

The reliability of structure is often affected by the material, stress, geometric shape and the size of structure, and these parameters are uncertain. Therefore, in reliability analysis, these parameters are usually called the basic random variables. Suppose there are *N* random variables, and they are usually expressed as $x_i(i=1,2,\ldots,n)$. Using $g(x)$ to express the function of structure [10]:

$$Z = g(x) = g(x_{1,} x_{2,} \cdots, x_n) \tag{1}$$

in which, the value of *Z* determines the state of the structure. When *Z* is greater than *0*, the structure is in a reliable state; when *Z* is less than *0*, the structure or machine is in a failure state; when *Z* is equal to *0*, the structure is in a limit state. $Z = g(x) = 0$ is called the limit state equation, and the surface generated in the coordinates is called the limit state surface, which is also called the failure surface. When the function of the structure is only related to the load effect *S* and the structural resistance *R*, the function can be written as $Z = (R,S) = R - S$, and the structure is in the same state as above.

Structural reliability indicates the probability that a mechanism can perform a predetermined operation function under the life cycle and the use environment. It is commonly used (Multiple integrals of the joint density function of the variable *X* in the reliable domain Ω_s of f_x) by p_r is expressed:

$$R = \mathrm{P_r} = \int_{\Omega_s}\cdots\int f_x d_x \tag{2}$$

The failure probability indicates the probability that an organization will fail to perform its intended function in the life cycle and use environment. It is commonly used (Multiple integrals of the joint density function of the variable *X* in the reliable domain Ω_f of f_x) P_f is expressed:

$$\mathrm{P}_f = \int_{\Omega_f}\cdots\int f_x d_x \tag{3}$$

Reliability and failure probability are necessarily complementary, and their relationship is:

$$\mathrm{P}_s + \mathrm{P}_f = 1 \tag{4}$$

Reliability index: Using R-F (Lakovitz-Ficeley method) to transform some non-normal variables into the standard equivalent normalization, we can obtain the normal distributed variables, such as the mean σ'_{xi}, the standard deviation μ'_{xi}, and the reliability index β. They are respectively given as

$$\begin{aligned} &\sigma'_{xi} = \phi\left\{\Phi^{-1}\left[F_{\mathrm{xi}}\left(xi^*\right)\right]\right\} / f_{xi}\left(X_i^*\right) \\ &\mu'_{xi} = X_i^* - \Phi^{-1}\left[F_{xi}\left(xi^*\right)\right] / \sigma'_{xi} \\ &\beta = \left(\sum_{i=1}^{n}\left[\left(X_i^* - \mu'_{xi}\right) / \sigma'_{xi}\right]^2\right)^{1/2} \end{aligned} \tag{5}$$

We take β as the shortest distance from the origin of the coordinates to the limit state surface. The corresponding point on the limit state surface is the design point or the check point. The mathematical model is expressed as

$$\begin{aligned} &Min\beta^2 = \sum_{i=1}^{n}\left[\left(X_i^* - \mu'_{xi}\right)/\sigma'_{xi}\right]^2 \\ &s.t. Z = g\left(X_1^*, X_2^*, \ldots, X_n^*\right) = 0 \end{aligned} \tag{6}$$

2.2. Augmented Multiplier Method

The penalty function method is a continuous approximation method that can be applied to most optimization problems. At the same time, the penalty function method can be combined with most unconstrained optimization algorithms, but the disadvantage is that the convergence rate is too slow, and the initial penalty factor has a greater impact on the solution result, so we use the augmented multiplier method. The augmented multiplier method introduces the multiplier term and the penalty term, which makes the convergence faster, and there are no excessive requirements on the value of the penalty factor. It only needs to take a larger penalty factor, or it can increase according to a certain ratio. We firstly establish an equality constraint model [11]:

$$\begin{aligned} &\min f(x), x \in R^n \\ &s.t. h_v(x) = 0, v = 1, 2, \cdots, n \end{aligned} \tag{7}$$

The Lagrangian multiplier transforms the equality constraint of Eq. (7) into an unconstrained optimization problem:

$$\begin{aligned} \min l(x,\lambda) &= f(x) + \lambda h(x) \\ &= f(x) + \sum_{v=1}^{M} \lambda_v h_v(x) \end{aligned} \tag{8}$$

The Lagrangian multiplier method is used to construct an unconstrained function， it is expressed as

$$\begin{aligned} M(x,\lambda,r) = f(x) + \frac{r}{2}\sum_{v=1}^{M}\left[h_v(x)\right]^2 \\ + \sum_{v=1}^{M} \lambda_v h_v(x) \end{aligned} \tag{9}$$

For the right side of Eq. (9): the first term is the objective function, the second term is the penalty term, and the third term is the multiplier term. $h_v(x)$ is the constraint function.

3. The immune algorithm

Immune algorithm is an intelligent optimization algorithm with high robustness, adaptability, global convergence, and parallel search, which is simulated based on the biological immune system and the working principle of gene evolution. It uses itself to generate the diverse populations, and evaluates the antibody concentration before each population refresh. And it suppresses the individuals with excessive concentrations, thereby can ensure the individual diversity. The immune algorithm evaluates the concentration of the antibody through the calculation of the affinity, and then respectively gives the antibody a greater incentive for the higher affinity and the lower concentration, and inherits the superior individual from the previous generation to the next generation through the cloning operator and the mutation operator. At the same time, the individuals with lower incentives are deleted to achieve the purpose of population update and the global optimal search. Its algorithm corresponds to one-to-one with the immune system, as shown in Table 1:

Table 1 The comparison between immune algorithm and immune system

Biological immunity	Immune algorithms
Antihelion	Optimization problem
Antibody(B cells)	The feasible solution to the optimization problem
Affinity	The mass of the feasible solution
Cell activation	Immune selection
Cell differentiation	Individual cloning
Affinity maturity	Variation
Clone inhibiting	Clone inhibiting
Dynamic balance	Population refresh

The evaluation formulas of the most important operators in the immune algorithm [12] are given as follows:

The affinity operator is given as

$$aff(ab_i, ab_j) = \sqrt{\sum_{K=1}^{L}(ab_{i,k} - ab_{j,k})^2} \tag{10}$$

The concentration evaluation operator is provided as

$$den(ab_i) = \frac{1}{N}\sum_{j=1}^{N} S(ab_i, ab_j) \tag{11}$$

The incentive calculation operator is given as

$$sim(ab_i) = aff(ab_i) \bullet e^{-a \cdot den(ab_i)} \tag{12}$$

in which, ab_i represents the i -th antibody in the population, and $ab_{i,k}$ represents the k -th dimension of the D-th antibody.

The iterative process is shown in Fig. 1.

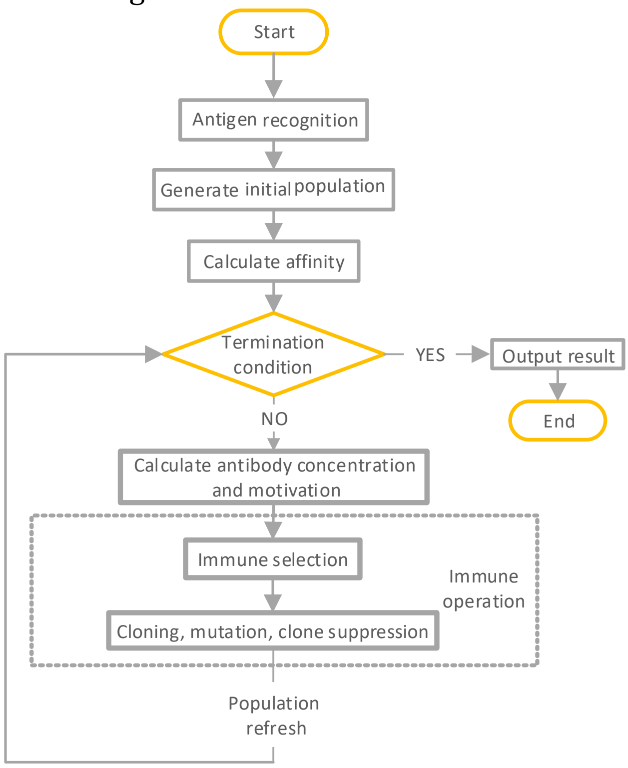

Fig. 1 The flowchart of Immune algorithm

4. . Numerical examples

Consider a set of limit state equations which consist of variables x_1, x_2 as follows:

$$g(x) = x_1 - 1.7x_2 + \alpha(x_1 + 1.7x_2)^2 + 5 \quad (13)$$

in which, x_1 and x_2 obey the standard normal distribution, and the value of α determines the degree of nonlinearity of the limit state equation. Let α be 0.015. Using the proposed algorithm in this paper, the penalty factor R is 1000, the number of chromosomes proposed is NP = 50, the maximum genetic generation number is the end condition of the operation is 10 generations, the probability of mutation P_m = 0.7, the incentive coefficient φ = 1, and the similarity threshold δ = 0.2, number of clones N_{cl} = 10, the results are given as follows:

(1) The goal of the proposed method is 10 iterations, the third time has been converged. The MPP point is (-1.5137, 2.0867), and the reliability index is β = 2.5779. The failure rate is P_f = 0.0050.

(2) Optimal search of the objective function through the gradients using FOSM. The objective function converges at the 8th time. The MPP point is (-1.4362, 2.1391). The reliability index is β = 2.5765, and the failure rate P_f = 0.0050.

(3) MCS is used to perform 1 × 10^7 sampling simulations on the objective function. The reliability index β = 2.6011 and the failure rate P_f = 0.0046.

The computational results of the proposed method in this paper are shown in Table 2 and Table 3:

Table 2 Iterative process of the proposed method in this paper

Number of iterations	Reliability index β	Failure probability P_f
1	2.5441	0.0055
2	2.7071	0.0034
3	2.5779	0.0050
4	2.5779	0.0050
5	2.5779	0.0050
6	2.5779	0.0050
7	2.5779	0.0050
8	2.5779	0.0050
9	2.5779	0.0050
10	2.5779	0.0050

Table 3 The comparison of three methods

Method	Reliability index β	Failure probability P_f	Number of convergence
MCS	2.6011	0.0046	10^7
FOSM	2.5765	0.0050	8
The proposed method	2.5779	0.0050	3

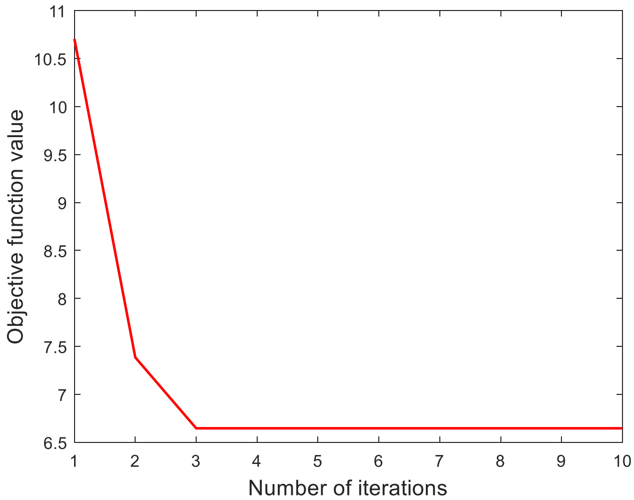

Fig. 2 The convergence process of the proposed method

According to Table 2, Table 3, and Fig. 2, it can be seen that the proposed method in this paper converges faster than the FOSM method. The values of the reliability index and the failure probability obtained by the proposed method are similar to the MCS method and the FOSM method, and its accuracy is high. The proposed method in this paper is practical and reliable, and can effectively avoid the selection of penalty factors and the difficulty in derivation in engineering cases.

In order to study the stability of the proposed algorithm for solving the nonlinear limit state equation in this paper, the value of α respectively takes 0.15, 0.2, 0.25, 0.35, and the solution of the proposed method is adopted. The results are shown in Table 4:

Table 4 The computational results under different degrees of nonlinearity

Parameter α	Method	Reliability index β	Failure probability P_f	Number of convergence
0.15	MCS	2.9289	0.0017	10^7
	The proposed method	2.7525	0.0030	4
0.2	MCS	2.9912	0.0014	10^7
	The proposed method	2.7867	0.0027	7
0.25	MCS	3.0376	0.0012	10^7
	The proposed method	2.7973	0.0026	7
0.35	MCS	3.0975	9.7580e-04	10^7
	The proposed method	2.8202	0.0024	8

According to Table 4, it can be seen that the proposed method is used to solve the problem under different degrees of nonlinearity, and the computational results are similar to those obtained by the MCS method, while the FOSM method cannot solve this problem. The proposed method in this paper is very reliable and effective, and has good stability and convergence

5. Engineering application

The reliability-based optimization design of cantilever beams, the structure is shown in Fig. 3. The arm length is respected by L, the cross section height is denoted by h, and the width is b. The top of the cantilever beam bears forces P_x and P_y from the horizontal and vertical directions respectively, and the fixed end on cantilever beam cannot exceed its yield strength limit $S = 370MPa$, and the limit state equation [13] can be expressed as

$$g(b,h,L,P_x,P_y) = S - \frac{6P_xL}{b^2h} - \frac{6P_yL}{bh^2} \tag{14}$$

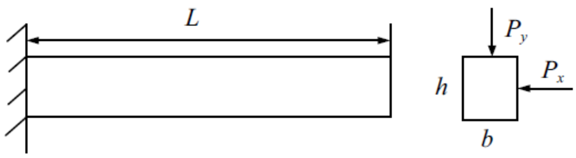

Fig. 3 Schematic diagram of cantilever structure

where the distribution types and parameters of each variable are shown in Table 5.

Table 5 The relationship between random variables

Parameter	Mean μ_x	Standard deviation σ_x	Distribution type
$b\,/\,mm$	100	15	Normal
$h\,/\,mm$	200	20	Normal
$L\,/\,mm$	2500	300	Lognormal
parameter	lower limit	lower limit	Distribution type
$P_x\,/\,N$	47000	53000	Interval
$P_y\,/\,N$	23000	27000	Interval

Calculated by this method, its MPP point is (112.6， 189.9， 2287.2， 49454， 26050), reliability index β = 1.7402, the failure rate P_f = 0.0409, and the iteration curves of reliability β and failure probability P_f are given as follows:

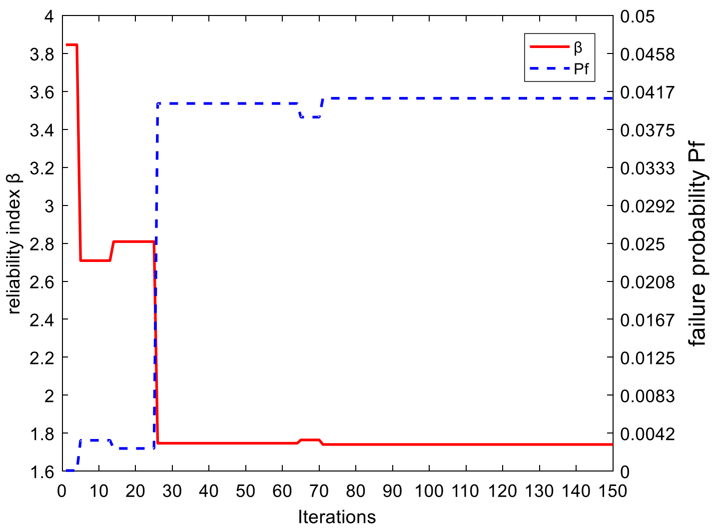

Fig. 4 Reliability index iteration curve

According to Fig. 4, the method in this paper converges basically in the 30th iteration, then the numerical value tends to be stable, and converges completely in the 70th iteration. Therefore, this method has fast convergence speed and high accuracy, and has good effect on nonlinear problems.

However, in practical engineering, there is often a certain relationship between variables. Therefore, this paper introduces the correlation coefficient to do the correlation analysis. The reliability index is given by (16). Considering the positive correlation between the height h and the width b of the cross section. The correlation coefficients are respectively given by $\rho_{bh} = 0.4 \sim 0.8$.

$$\beta = \frac{a_0 + a_1\mu_{X_1} + \cdots + a_n\mu_{X_n}}{\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} a_i\sigma_{X_i}\rho_{X_iX_j}\sigma_{X_j}a_j}} \tag{16}$$

The proposed method is used to solve these cases, and the results are shown in Fig. 5.

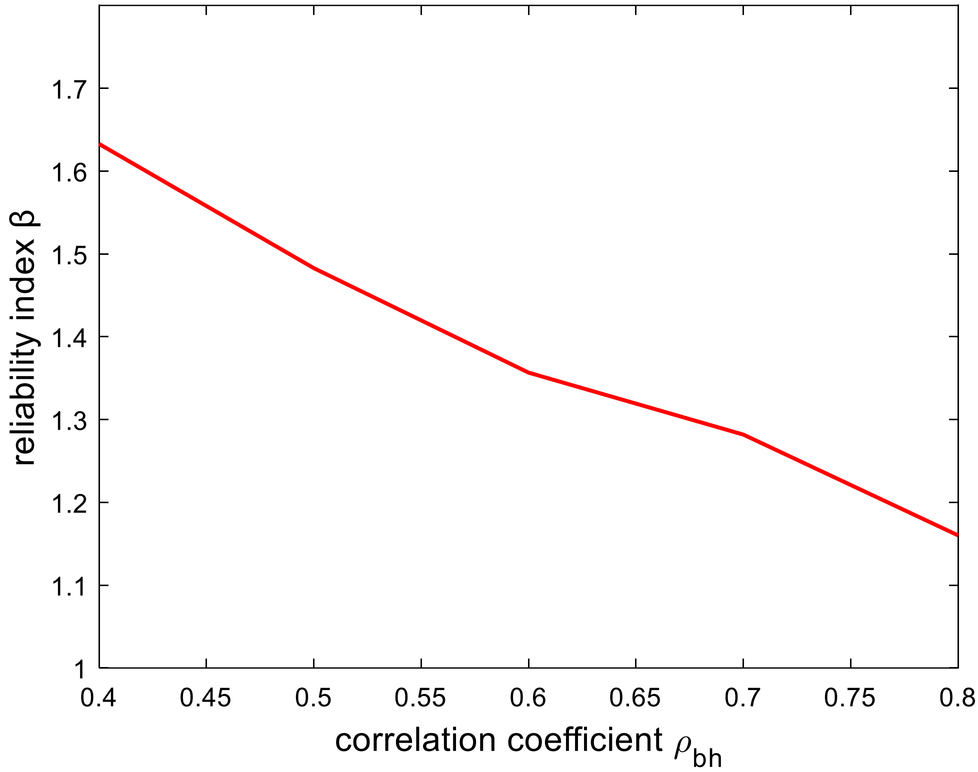

Fig. 5 The relationship between the correlation coefficient and the reliability index

According to Fig. 5, with the gradual increase of $|\rho_{bh}|$, the reliability index β is decreasing, so the correlation between variables b and h should be paid more attention in engineering design.

6. Conclusion

This paper proposes a new hybrid reliability analysis algorithm based on augmented multiplier method and immune algorithm, and it provides another way to deal with practical reliability engineering problems. Firstly, the minimum reliability index is used as the objective function, and the structural limit state equation is used as a constraint. They are used to establish the mathematical model of reliability optimization design. The immune algorithm is used to solve the optimization model, and the optimal design point is obtained. Numerical and engineering examples are investigated to verify the proposed algorithm in this paper. The results show that the proposed method in this paper has good convergence and good accuracy in solving practical reliability engineering problems.

Acknowledgements

This work is supported by the Open Fund of Hubei key Laboratory of Hydroelectric Machinery Design and Maintenance (2019KJX12).

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