Reliability Analysis Method Based on Augmented Multiplier Method and Firefly Algorithm

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Abstract

This paper proposes a reliability analysis method based on the hybrid algorithm of firefly algorithm and augmented Lagrange multiplier method. The algorithm takes the performance function composed of random variables affecting the structure as the constraint, and the minimum reliability index as the mathematical model. The mathematical model constraint is transformed into an unconstrained model by the augmented Lagrange multiplier method, and then the firefly algorithm (FA) is used for iterative optimization. Through engineering examples and numerical examples, the proposed algorithm is compared with the first-order second-moment method and Monte Carlo method to verify the effectiveness and high efficiency of the proposed algorithm.

Keywords

Augmented Lagrange multiplier method; Reliability analysis; Firefly algorithm.

1. Introduction

In practical engineering, due to the combined action of multiple uncertain factors such as load, service environment and internal factors of materials, the reliability of structures will decrease with the increase of time. For example, tool wear will affect the machining accuracy, and material quality will affect various parameters of materials. Therefore, in order to ensure normal work, a large number of scholars have proposed methods to calculate the reliability [1-3]. Jiang et al. [4] proposed an AK-MCS method of hybrid particle swarm optimization algorithm, which reduces the number of iterations of Kriging surrogate model while ensuring the accuracy. Zhang et al. [5] used artificial neural networks to fit limit state functions to solve reliability problems. Zhu et al. [6] proposed particle swarm optimization based harmonious search algorithm (PSO–HS) and enhanced particle swarm optimization (EPSO), which improved the convergence speed and global convergence ability of particle swarm optimization algorithm. Zhang et.al [7] presents an effective active-learning based Kriging method for structural reliability analysis. Huang et al. [8] proposed a hybrid reliability method of crowd search algorithm and augmented multiplier method based on the problem of insufficient accuracy in traditional optimization algorithms. Liao et al. [9] introduced parameter uncertainty and interval variables and proposed a reliability analysis method based on particle swarm optimization algorithm and augmented multiplier method. The augmented multiplier method can effectively transform constrained models into unconstrained models through Lagrangian analysis, making it easier to perform the reliability iterative search.

Among various classic reliability evaluation methods, there are still many limitations, such as the Monte Carlo method requiring a sufficient number of samples, but the more sample values, the greater the computational complexity; However, the first order second moment method is only applicable to linear or micro linear differential equations, and because the accuracy is affected by the step size, it is difficult to obtain the ideal accuracy if the step size is too long, and the error caused by the stochastic process cannot be eliminated even if the step size is too small. The mixed reliability analysis based on the augmented multiplier method and the Firefly algorithm proposed in this paper can transform the reliability constraint model into an unconstrained model, avoiding the selection of penalty factors. Moreover, the Firefly algorithm has high precision and fast iteration, and can quickly find the optimal value.

2. Reliability Theory

Suppose that there are n random variables $X=(X_1, X_2, \ldots, X_n)$ that affect the structural function, so the function of random variables is called :

$$Z=g(X)=g(X_1, X_2, \ldots, X_n) \tag{1}$$

It is called the performance function or limit state function of the structure. When indicates that the structure is reliable ; at the same time, $Z=0$ represents the limit state equation of the structure in the random variable space, which divides the definition domain into the reliability domain and the failure domain. At the same time, the reliability index of the structure is defined by the ratio of the mean value and the standard deviation of the random variable. The larger the reliability index, the more reliable the structure is. According to the law of large numbers, the Monte Carlo method (MC) samples the random variables and brings the sampling points into the performance function. If $Z<0$ represents a failure, the corresponding failure probability is obtained according to the ratio of the total number of failures to the number of sampling points. The smaller the failure probability is, the more reliable it is to measure the reliability of the structure.

The Rackwitz-Fiessler method (JC method) in the first-order second-moment method uses the following formula to update the mean and standard deviation of random variables [10]:

$$\mu'_{xi}=X_i^*-\Phi^{-1}[F_{xi}(x_i^*)]\sigma'_{xi} \tag{2}$$

$$\sigma'_{xi}=\frac{\phi\{\Phi^{-1}[F_{xi}(x_i^*)]\}}{f_{xi}(x_i^*)} \tag{3}$$

where $F_{xi}(\cdot)$ and $f_{xi}(\cdot)$ are the cumulative distribution function and probability density function of random variables respectively, the reliability index is defined as

$$\beta=\sqrt{\sum_{i=1}^{n}[(X_i^*-\mu'_{xi})/\sigma'_{xi}]^2} \tag{4}$$

where X^* represents the point on the limit state equation, the minimum value of β can be obtained, and the minimum value represents the reliability of the structure under the constraint that the performance function is zero. The corresponding mathematical model is given as follows:

$$\begin{aligned} &\min M(X)=\beta^2=\sum_{i=1}^{n}[(X_i^*-\mu'_{xi})/\sigma'_{xi}]^2 \\ &s.t. Z=g(X_1^*, X_2^*, \ldots, X_n^*)=0 \end{aligned} \tag{5}$$

The above model has constraint, which can be converted into an unconstrained model by the augmented Lagrange multiplier method:

$$\min M_{\lambda}(X^*, r, \lambda) = M(X^*) + \frac{r}{2}[g(X^*)^2] + \lambda g(X^*) \tag{6}$$

where M_{λ} is the fitness of the unconstrained optimization model, λ is the Lagrange multiplier, λ is the multiplier term, r is the penalty term, and $M(X^*)$ is the objective function. Using the augmented multiplier method, it is only necessary to take the penalty factor as a relatively large number to increase proportionally.

3. Firely Optimization Algorithm

The firefly algorithm is an optimization algorithm based on the glowing swarm behavior of fireflies. The algorithm is based on the glowing characteristics of fireflies to find partners in the search space, and moves toward the individual with the strongest glowing, so as to realize the position optimization update. In the optimization calculation, it has the following characteristics. The attraction between fireflies is related to the position. Individuals will compare with each other. The weak light will think about the movement of light intensity. At the same time, the individual attraction of light intensity is also related to the distance. The farther the distance is, the weaker the attraction is. In the specific problem, the light intensity and the moving area are determined by the adaptive function. The firefly algorithm is simple and efficient, and the iteration speed is fast.

3.1. Fluorescence Intensity

The fluorescence intensity of individual fireflies is determined by the following formula:

$$\min M_{\lambda}(X^*, r, \lambda) = M(X^*) + \frac{r}{2}[g(X^*)^2] + \lambda g(X^*) \tag{7}$$

In the formula, I_0 represents the brightness of the firefly itself, which is related to the objective function, that is, the better the objective function value, the higher the brightness; γ is the absorption coefficient of light intensity. Because the light intensity will decrease with the increase of distance and the absorption of the media, $\gamma \in [0.01, 100]$ is usually set. r is the Euclidean distance between firefly individuals :

$$r_{ij} = \left\| x_i - x_j \right\| = \sqrt{\sum_{k=1}^{d} (x_{i,k} - x_{j.k})} \tag{8}$$

where d is the spatial dimension, and $x_{i,k}$ is the k -th component of the firefly in the d -dimensional space.

3.2. Attractiveness

$$\alpha(r) = \alpha_0 e^{-\gamma r^m}, (m \geq 1) \tag{9}$$

where α_0 is the maximum attractiveness; the parameter m usually takes 2.

3.3. Location Update

By the firefly tending to the individual with higher brightness, the algorithm position update formula is given as follows:

$$x_i(t+1) = x_i(t) + \alpha(x_j(t) - x_i(t)) + \lambda \varepsilon_i \tag{10}$$

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In the formula, $x_i(t+1)$ is the new position of firefly i after the $t+1$-th movement ; λ is the step factor, satisfying $\lambda \in [0,1]$; ε_i is a random factor satisfying Gaussian distribution in $[0,1]$ interval. The flow chart of the algorithm is given as follows：

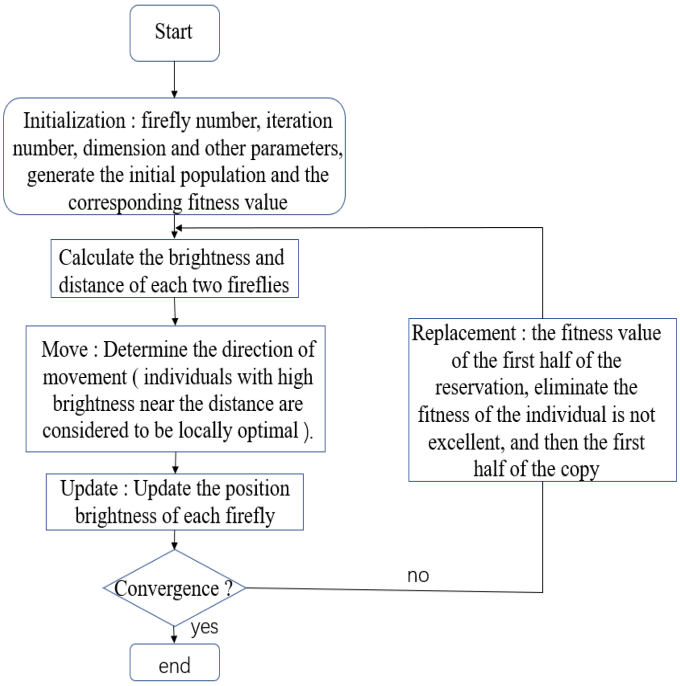


Figure 1 The computation flow chart of the Firefly algorithm

4. Numerical examples

Suppose that there are random variables x_1 and x_2 with mean $[10, 2.5]$ and standard deviation $[2, 0.375]$, and their performance functions satisfy:

$$g = 18.46 - 7.48 x_1 x_2^{-3} \tag{11}$$

In this algorithm, the penalty factor r is 1000, the population number b is 50, the dimension is 2, the maximum number of iterations is 20, the maximum attraction α_0 is 2, the light absorption intensity γ is 1, and the step factor λ is 0.2; after iterative optimization of the Firefly algorithm, the results are shown in Table 1 below. At the same time, this example is brought into the first second moment method and the Monte Carlo method. The comparison results are listed in the following table. The results show that the present method in this paper calls functions more times than the first second moment method, but less times than the Monte Carlo method; At the same time, the convergence speed of this method is faster than that of the first order second moment method, and the reliability index results are close to the two methods:

Table 1 The reliability index results of the three methods

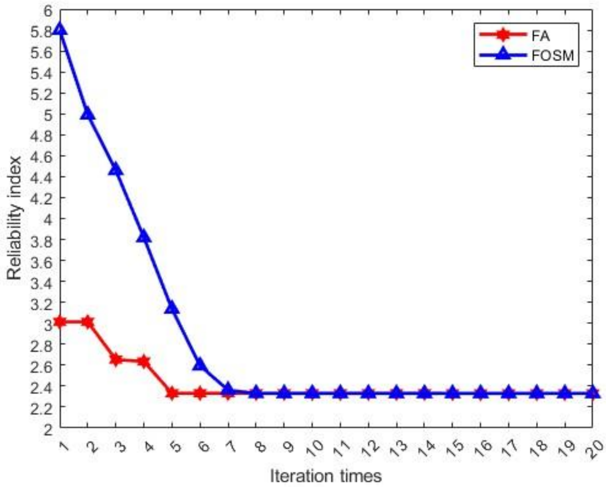
Methods	Reliability Index	Iterative Times	Number of function calls
MCS	2.3430	—	107
FOSM	2.3302	8	8
Method of this article	2.3324	5	40×50

The calculation results are shown in Table 2 and Figure 2. Based on the graph, it can be concluded that the present method in this article converged in step 5, which is faster than the first-order second-order moment method.

Table 2 The iterative calculation results of the present algorithm in this paper

Iteration times	β	P_f
1	3.0137	0.0013
2	3.0137	0.0013
3	2.6534	0.0040
4	2.6352	0.0042
5	2.3324	0.0098
6	2.3324	0.0098
7	2.3324	0.0098
8	2.3324	0.0098
9	2.3324	0.0098
10	2.3324	0.0098

Figure 2 Iterative results of FOSM algorithm and FA algorithm. At the same time, the reliability index calculated by this method is 2.3324, which is close to the results of FOSM and MCS methods. Therefore, the method proposed in this article has high accuracy while ensuring a certain level of efficiency, and the feasibility of the proposed method has been verified through comparison.



5. Engineering Example

An example of a rectangular simply supported beam is considered. The length of the simply supported beam is l, the width of the cross section is b, the height is h, and the uniform load q is applied throughout it. The strength is R, and all random variables obey the normal distribution. The material is 45 steel. The parameters of the simply supported beam are shown in Table 3, and the diagram is shown in Fig. 3. The performance function is the difference between strength and load. When the strength ratio is small, the structure is considered unreliable:

$$Z = G(q, R, l, b, h) = R - \frac{0.75 \times q \times l^2}{b \times h^2} \tag{12}$$

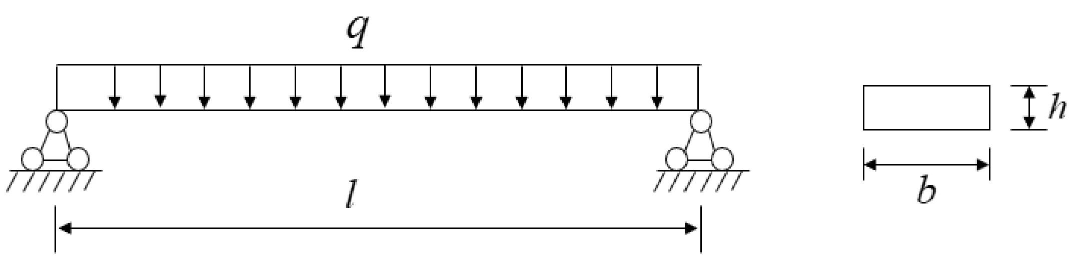


Fig. 3 Simple supported beam structure diagram

Table 3 Distribution types and parameter values of uncertain variables of simply supported beams

Parameter	Mean μ_x	Standard deviation σ_x	Distribution type
l / mm	4000	900	Normal
b / mm	120	30	Normal
h / mm	240	50	Normal
R / MPa	0.5	0.1	Normal
$q / (N / mm^2)$	210	50	Normal

Optimization iteration by firefly algorithm, its MPP point is (1300, 210, 390, 0.8, 60), reliability index β = 6.7082, failure rate $P_f = 9.8\times10^{-12}$, the convergence diagram of reliability index and failure probability is shown in Figure 4.

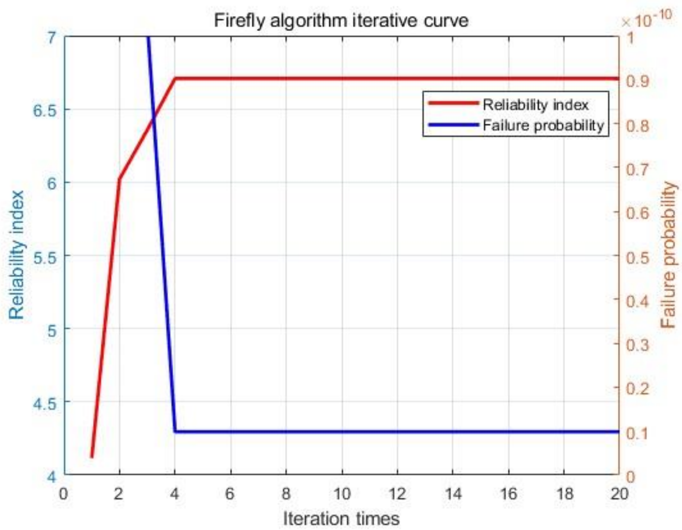


Fig. 4 Reliability index and failure probability iterative curve

It can be seen from Figure 4 that the reliability index converges to the fourth step, and the failure probability converges in the fifth step. Therefore, it can be seen that the firefly algorithm has strong local search ability, can find the optimal solution in a very small area, and has high precision.

6. Conclusion

This paper presents a hybrid reliability analysis method of augmented Lagrange multiplier method and Firefly algorithm. The reliability mathematical model calculates the minimum reliability index with the limit state function as the constraint. This method converts the reliability function constraint into an unconstrained model by using the augmented multiplier method, and then uses the Firefly algorithm to solve it. The feasibility of the method in this paper is verified by comparing the numerical example with the first order second moment method and the Monte Carlo method. At the same time, an engineering example is used, and this indicates that the proposed method has good convergence speed and robustness in solving practical engineering problems.

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