# Bluetooth Indoor Positioning Research Based on RSSI of the Least Square Positioning Algorithm

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#### **Abstract**

The biggest advantage of Bluetooth indoor positioning technology is equipment smaller, which can be integrated in the handheld devices, personal computers and mobile terminal equipment. This paper first introduces the evaluation standards of indoor positioning performance, then expounds the characteristics of Bluetooth signal and the weighted least square positioning algorithm of eliminating high order based on Received Signal Strength Indicator(RSSI), and finally summarizes the full text.

# **Keywords**

Bluetooth indoor positioning technology, Bluetooth signal, least square positioning algorithm, RSSI.

#### 1. Introduction

Indoor positioning technology uses indoor correlated wireless signal, and combines indoor localization method of implementing indoor location technology. Wireless local area network (LAN) is the base of the Bluetooth positioning technology, deployed proper number of Bluetooth access points in the region of the positioning. When positioning, LAN is configured to basic network architecture, the Bluetooth labels and access points accessed to LAN proceed two-way communication, and through the correlation algorithms estimates the labels' position. It can be positioning by measuring the value of signal strength Received Signal Strength Indicator(RSSI). The shortcomings of low version of the Bluetooth positioning are poor stability, susceptible to noise interference, and is only suitable for short distance, small range of positioning.

## 2. The evaluation standards of indoor location performance

In indoor localization algorithms, researchers mostly concern about the positional accuracy and positional performance of the robustness of the algorithm, positional cost and the deployment of complexity, etc. The main performance indexes of indoor localization algorithm are under discussion in the following.

#### 2.1 The minimum and maximum error

The minimum error is the minimum value of the positioning error required by the localization algorithm in simulation experiments, which means the localization algorithm or positioning system can achieve the best positioning effect. The maximum error is the maximum value of the positioning error required by the localization algorithm, which reflects the localization algorithm or positioning system of the worst situation.

#### 2.2 The complexity

With the operation time(calculated quantity) of the localization algorithm as the reference point, because the different performance of the mobile terminal CPU's operational capability and power consumption, it is very important to measure the complexity of algorithm. Not only should consider mobile terminal memory's storage size, but also attaches great importance to how much power loss

of the localization algorithm. Even if make the server bear all the positional calculation, but considering under the premise of multi-user sending out the requirements of positioning at the same time, and whether the positioning system can always stay safe and run smoothly and efficiently.

#### 2.3 MSE and RMSE

Mean Squared Error(MSE) and Root Mean Squared Error(RMSE) are two kinds of the common standards of evaluating positioning precision in wireless positioning system, the calculation method is as follows.

$$MSE = E \left[ \left( x - \hat{x} \right)^{2} + \left( y - \hat{y} \right)^{2} \right] = \frac{1}{n} \sum_{i=1}^{n} \left[ (x_{i} - \hat{x}_{i})^{2} + (y_{i} - \hat{y}_{i})^{2} \right]$$
(1)

$$RMSE = E\left[\left(x - \hat{x}\right)^{2} + \left(y - \hat{y}\right)^{2}\right] = \sqrt{\frac{1}{n} \sum_{i=1}^{n} \left[\left(x_{i} - \hat{x_{i}}\right)^{2} + \left(y_{i} - \hat{y_{i}}\right)^{2}\right]}$$
(2)

There (x, y) is a true coordinate position of the mobile terminal, and (x, y) is as the estimate of mobile terminal location coordinates. Usually used to the two evaluation criterion, namely MSE and RMSE, to compare, judge positioning performance of the various estimators.

#### 2.4 CDF

Cumulative Distribution Function(CDF) does the cumulative distribution of each coordinate estimating error. Through this function, it can be very intuitive to see the distribution probability under a certain value of position error. The formula of CDF is:

$$CDF(e_i) = \frac{\sum_{i=1}^{i} e_i}{\sum_{i=1}^{n} e_i}$$
(3)

There into,  $\sum_{i=1}^{i} e_i$  is positioning error of error sampling points number under the scope of  $e_i$ , and

 $\sum_{i=1}^{n} e_i$  is the total number of error sample points. In the design and application of the algorithm and system, generally need to set the value of CDF, so that it is less than or equal to a certain value.

### 3. Ranging based on RSSI

The ranging principle based on received signal strength indicator(RSSI) is that a radio signal will happen loss of energy intensity in propagation process, and the wastage of the radio signal's energy intensity and the transmission distance meet certain mathematical function relationship, through the mobile terminal to collect the value of RSSI around each beacon node launching, using the path loss model to make the value of RSSI be converted to the distance from mobile terminal to each beacon node.

### 3.1 The characteristics of Bluetooth signals

The localization algorithm based on RSSI needs to analyze the characteristics of Bluetooth signals. The carrier frequency of Bluetooth signals has only 2.4GHz, Bluetooth realizes the expansion of the spectrum using frequency modulation method, to divide the 2.4 GHz frequency band into 79 frequency points. The data transmission of Bluetooth beacon node is more complex, first sending data at a certain frequency point, and then jump to another frequency point to continue sending data, while these data transmission of frequency point arrangement is pseudo random, frequency shift 1600 times per second, and the duration of each frequency is 625us.

The protocol stack of Bluetooth specification is the protocol for the personal area of designing and establishing wireless communication, using the hierarchical structure to respectively complete the functions of data flow transmission and filtering, the establishment and release of wireless connection, the guarantee of the quality of business services, the control of the link, the demultiplexing and multiplexing of the protocol, etc. The stack of Bluetooth protocol follows the reference model of open systems interconnection, as shown in Fig. 1, and from high to low Bluetooth protocol can be divided into three layers, namely, the application layer protocol suit(ALPS), the middleware protocol suit(MPS), the transport layer protocol suit(TLPS).

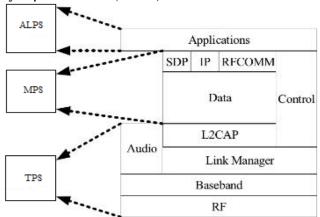


Fig. 1 The protocol stack for Bluetooth

#### 3.2 The weighted least square positioning algorithm of eliminating high order based on RSSI

#### Preprocessing signal parameters

In the positioning process of mobile terminal, terminal receives N RSSI values of certain beacon node, because this N RSSI values are discrete random variables, whose values subject to or similarly subject to Gaussian distribution. In order to get rid of the random and noise interference of signal acquisition process and improve the positioning precision of the positioning algorithm, generally choose those with high probability, that is, the area of high distribution density, measure the signal strength of beacon node about x's density function of f(x), as shown in equation 4:

$$f(x) = \frac{1}{\delta\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2\delta^2}} \tag{4}$$

There  $\mu$  and  $\delta$  are the average value and standard deviation of the signal intensity respectively, which can solve through the following formulation:

$$\begin{cases} \mu = \sum_{i=1}^{N} RSSI_{i} \\ \delta = \sqrt{\frac{1}{N-1} \left( \sum_{i=1}^{N} (RSSI_{i} - \mu)^{2} \right)} \end{cases}$$
 (5)

After all the values of the collected signal strength are to pass Gaussian filter, the range of the signal is within  $[0.15\delta + \mu, 3.09\delta + \mu]$  of the RSSI values. Assuming that remain M RSSI values after filtering, which is processed to be optimized. The processed RSSI values are as the RSSI values of beacon nodes that participate in the positioning of the positioning algorithm in this period of time.

The optimized estimation of pseudo-range

Assume the transmission power of beacon node is Pt, whose unit is dBm, according to Huawei indoor path loss model and the preprocessed M RSSI values, it can get the received signals of pending positioning nodes is:

$$RSSI = Pt - 20 \times \log(f) - 10 \times n \times \log(d) - \sum_{i=0}^{N} P_i - X_{\sigma} + 28$$
 (6)

Let  $A = Pt - 20 \times \log(f) - \sum_{i=0}^{N} P_i - X_{\sigma} + 28$ , and then the formulation of (6) can be simplified:

$$RSSI = A - 10 \times n \times \log(d) \tag{7}$$

M RSSI values can be obtained the equation set:

$$\begin{cases}
A - RSSI_1 = 10 \times n \times \log(d_1) \\
A - RSSI_2 = 10 \times n \times \log(d_2) \\
& \cdots \\
A - RSSI_M = 10 \times n \times \log(d_M)
\end{cases}$$
(8)

The above formulation can be abbreviated to the form below:

$$q\theta = b \tag{9}$$

There into,  $q = 10 \times n$ ,  $\theta = [\lg d_1, \lg d_2, \dots, \lg d_M]^T$ ,  $b = [(A - RSSI_1), (A - RSSI_2), \dots, (A - RSSI_M)]^T$ . Using the least square method to resolve the above formulation to get:

$$\hat{\theta}_{LS} = (q^T q)^{-1} q^T b \tag{10}$$

$$\hat{d} = 10^{\hat{\theta}_{LS}} \tag{11}$$

So it can get the distance from the receiver to the transmitter of least-square estimate value  $d_{LS}$ , as shown in equation 12:

$$\hat{d}_{LS} = \frac{1}{M} \sum_{i=1}^{M} \hat{d}_i \tag{12}$$

Here,  $\hat{d}_i$  is the *ith* element of the vector  $\hat{d}$ ,  $i = 1, 2, \dots, M$ . The least squares estimate values  $\hat{d}_{LS}$  is substituted into the circle equation, namely, to realize the RSSI value of the least squares estimated process.

3.2.3 The weighted least square positioning algorithm of eliminating high order based on RSSI According to the formulation (12), it can be calculated respectively the distance from mobile terminals to n launching beacon node  $d_i(i=1,2,3,\cdots,n)$ . Suppose the position coordinates of n launching beacon nodes are all known, it can obtained the observed circular equation, as shown in equation 13:

$$\begin{cases} (x_1 - x)^2 + (y_1 - y)^2 = d_1^2 \\ (x_2 - x)^2 + (y_2 - y)^2 = d_2^2 \\ \cdots \\ (x_n - x)^2 + (y_n - y)^2 = d_n^2 \end{cases}$$
(13)

There into,  $(x_i, y_i)(i = 1, 2, 3, \dots, n)$  is known to the coordinates of launching beacon nodes, (x, y) is the mobile terminal's coordinates.

The method of eliminating higher order is through calculating the difference between the observation equation to eliminate secondary unknown parameters of observation equation. The formulation (13) of the *nth* equation as the benchmark, the rest equations are all subtracted with it, and it can be got a new observation equation:

$$\begin{cases}
d_1^2 - d_n^2 = x_1^2 + y_1^2 - (x_n^2 + y_n^2) - 2(x_1 - x_n)x - 2(y_1 - y_n)y \\
d_2^2 - d_n^2 = x_2^2 + y_2^2 - (x_n^2 + y_n^2) - 2(x_2 - x_n)x - 2(y_2 - y_n)y \\
& \cdots \\
d_{n-1}^2 - d_n^2 = x_{n-1}^2 + y_{n-1}^2 - (x_n^2 + y_n^2) - 2(x_{n-1} - x_n)x - 2(y_{n-1} - y_n)y
\end{cases} (14)$$

To transform the formulation equations (14) can be obtained the form such as  $V_e = B_e Z_e - L_e$  of the error equation:

$$\begin{bmatrix} v_{1} \\ v_{2} \\ \dots \\ v_{n-1} \end{bmatrix} = \begin{bmatrix} 2(x_{1} - x_{n}), 2(y_{1} - y_{n}) \\ 2(x_{2} - x_{n}), 2(y_{2} - y_{n}) \\ \dots \\ 2(x_{n-1} - x_{n}), 2(y_{n-1} - y_{n}) \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} x_{1}^{2} + y_{1}^{2} - (x_{n}^{2} + y_{n}^{2}) + d_{n}^{2} - d_{1}^{2} \\ x_{2}^{2} + y_{2}^{2} - (x_{n}^{2} + y_{n}^{2}) + d_{n}^{2} - d_{2}^{2} \\ \dots \\ x_{n-1}^{2} + y_{n-1}^{2} - (x_{n}^{2} + y_{n}^{2}) + d_{n}^{2} - d_{n-1}^{2} \end{bmatrix}$$

$$(15)$$

For the linear formulation equations  $V_e = B_e Z_e - L_e$ , using the method of the least square estimates the estimated value of the state variable  $Z_e$  from the observation vector  $L_e$ ,  $V_e$  can be understood as the observation vector  $L_e$  of the error vector. The estimate criterion of the weighted least squares is to make the sum of weight error of squares:

$$J\left(\hat{Z}_{e}\right) = \left(L_{e} - B_{e} \hat{Z}_{e}\right)^{T} W_{e} \left(L_{e} - B_{e} \hat{Z}_{e}\right) \tag{16}$$

achieve a minimum.  $W_e$  is a positive definite and diagonal weighted matrix, under the conditions of the ratio of ranging error and the distance as the independent distributed Gaussian random variable, it can be resolved by ranging error variance matrix  $V_e$ , and  $W_e = V_e^{-1}$ . The distance between each launching beacon node and the mobile terminal  $d_i$  ( $i = 1, 2, 3, \dots, n$ ) has an impact on the positioning accuracy, short distance has bigger influence on the position precision and long distance has little effect on the positioning accuracy. The influence of the different distance on the positioning accuracy is also reflected in  $V_e$  matrix, its form is:

$$V_{e} = \begin{bmatrix} d_{1} - d_{n}, 0, \cdots, 0 \\ 0, d_{2} - d_{n}, \cdots, 0 \\ \cdots, \cdots, \cdots, \cdots \\ 0, 0, \cdots, d_{n-1} - d_{n} \end{bmatrix} \begin{bmatrix} \sigma_{1}^{2}, 0, \cdots, 0 \\ 0, \sigma_{2}^{2}, \cdots, 0 \\ \cdots, \cdots, \cdots, \cdots \\ 0, 0, \cdots, \sigma_{n-1}^{2} \end{bmatrix} \begin{bmatrix} d_{1} - d_{n}, 0, \cdots, 0 \\ 0, d_{2} - d_{n}, \cdots, 0 \\ \cdots, \cdots, \cdots, \cdots \\ 0, 0, \cdots, \sigma_{n-1}^{2} - d_{n} \end{bmatrix}$$

$$(17)$$

Here,  $\sigma_1^2, \sigma_2^2, \dots, \sigma_{n-1}^2$  is the sum between the distance  $d_i$  ( $i = 1, 2, 3, \dots, n-1$ ) and  $d_n$  of the variance, because the distance between each launching beacon nodes and mobile terminal is mutually independent. The matrix  $V_e$  not only contains the distance to affect weight, but also contains the distance of different variance impacted on weight.

Let  $J\!\!\left(\stackrel{\widehat{Z}_e}{Z_e}\right)$  be derivation, and order:

$$\frac{\partial J(\hat{Z_e})}{\partial \hat{Z_e}} = 0 \tag{18}$$

So it can be obtained the solution of the estimated parameter  $Z_e = [x, y]^T$ , as shown in equation 19:

$$\overset{\wedge}{Z_e} = \begin{bmatrix} \overset{\wedge}{x} \\ \overset{\wedge}{y} \end{bmatrix} = (B_e^T W_e B_e)^{-1} B_e^T W_e L_e \tag{19}$$

#### 4. Conclusion

In this paper, the weighted least square positioning algorithm of eliminating high order based on Received Signal Strength Indicator(RSSI), using Bluetooth beacon nodes to localization, effectively improves the positional accuracy, which runs time significantly short, effectively enhances the efficiency of the positioning. In this article, Gaussian noise on the influence of the algorithm is smaller than the other algorithms, and is suitable for application in complex indoor environment.

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