An Approach to Evaluating Three-Dimension Reconstruction Image Quality with Hesitant Fuzzy Information

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Abstract

The problem of evaluating the three-dimension reconstruction image quality is the multiple attribute decision making (MADM) problems. In this paper, we investigate the multiple attribute decision making (MADM) problems for evaluating the three-dimension reconstruction image quality with hesitant fuzzy information. We utilize the hesitant fuzzy choquet ordered averaging (HFCOA) operator to aggregate the hesitant fuzzy information corresponding to each alternative and get the overall value of the three-dimension reconstruction image, then rank the three-dimension reconstruction images and select the most desirable one(s) by using the score functions of hesitant fuzzy values. Finally, an illustrative example for evaluating the three-dimension reconstruction image is given to verify the developed approach and to demonstrate its practicality and effectiveness.

Keywords

Multiple Attribute Decision Making(MADM), Hesitant Fuzzy Information, Hesitant Fuzzy Choquet Ordered Averaging (HFCOA) operator, Three-Dimension Reconstruction; Image Quality.

1. Introduction

Positron Emission Tomography (PET) acts as one of state-of-the-art technology of nuclear medicine. It provides tissue functional imaging with the aid of radioisotope, and can measure quantitative changes over time in the bio-distribution of radiopharmaceuticals throughout a target structure or the organs of interest. Physiological and/or biochemical parameters are then derived with the additional use of tracer kinetic modeling techniques. Positron emission tomography, however, is an ill-posed inverse problem because the observed projection data are contaminated by noise due to low count rate and physical effects. This problem of low counts is further accentuated with increased temporal sampling in a certain time period. Though traditional filter back projection (FBP) method has the advantage of less computation cost, it often results in noisy images of low quality. Better expressing system models of physical effects and modeling the statistical character of the measurement data, the maximum-likelihood expectation-maximization (ML-EM) approach outperforms the FBP method with regard to image quality and becomes the standard reconstruction algorithm instead of FBP for clinical PET. However, MLEM will produce increasing noise with the increasing iteration, thus results in non-convergence iteration. Recently, Bayesian methods or MAP (Maximum A Posteriori) methods solve this problem by incorporating image prior information and have been proved theoretically correct and practically effective compared to other methods. As to the problem of reconstructed image quality, Bayesian reconstruction can greatly improve reconstruction by incorporating prior information compared MLEM reconstruction. We also find that relied on the local neighborhood information of image-self, Bayesian methods can only contribute limit local prior information to reconstruction.

The problem of evaluating the three-dimension reconstruction image quality with hesitant fuzzy information is the multiple attribute decision making (MADM) problems [1-14].The aim of this paper is to investigate the MAGDM problems for evaluating the three-dimension reconstruction image quality with hesitant fuzzy information. Then, we utilize the hesitant fuzzy choquet ordered averaging

(HFCOA) operator to aggregate the hesitant fuzzy information corresponding to each alternative and get the overall value of the three-dimension reconstruction image quality, then rank the venture capital project and select the most desirable one(s) by using the score functions of hesitant fuzzy values. Finally, an illustrative example for evaluating the three-dimension reconstruction image quality is given to verify the developed approach and to demonstrate its practicality and effectiveness. The remainder of this paper is set out as follows. In the next section, we introduce some basic concepts related to hesitant fuzzy values. In Section 3 we introduce the multiple attribute decision making (MADM) problems to deal with appraisal model of three-dimension reconstruction image quality with hesitant fuzzy information, in which the information about attribute weights is correlative, and the attribute values take the form of hesitant fuzzy information. Then, we utilize the hesitant fuzzy choquet ordered averaging (HFCOA) operator to aggregate the hesitant fuzzy information corresponding to each alternative and get the overall value of the three-dimension reconstruction image quality, then rank the venture capital project and select the most desirable one(s) by using the score functions of hesitant fuzzy values. In Section 4, an illustrative example is pointed out. In Section 5 we conclude the paper and give some remarks.

2. Preliminaries

Atanassov[15-16] extended the fuzzy set to the IFS. However, when giving the membership degree of an element, the difficulty of establishing the membership degree is not because we have a margin of error, or some possibility distribution on the possibility values, but because we have several possible values. For such cases, Torra[17] proposed another generation of FS.

Definition 1[17]**.** Given a fixed set X , then a hesitant fuzzy set (HFS) on X is in terms of a function that when applied to X returns a sunset of $[0,1]$.

To be easily understood, Xu express the HFS by mathematical symbol:

$$E=\left(\left\langle x,h_E\left(x\right)\right\rangle \middle| x\in X\right), \tag{1}$$

where $h_E\left(x\right)$ is a set of some values in $[0,1]$, denoting the possible membership degree of the element $x\in X$ to the set E. For convenience, Xu call $h=h_E\left(x\right)$ a hesitant fuzzy element(HFE) and H the set of all HFEs.

Definition 2[18]. For a HFE h, $s(h)=\frac{1}{\#h}\sum_{\gamma\in h}\gamma$ is called the score function of h, where $\#h$ is the number of the elements in h. For two HFEs h_1 and h_2, if $s(h_1)>s(h_2)$, then $h_1>h_2$; if $s(h_1)=s(h_2)$, then $h_1=h_2$.

Based on the relationship between the HFEs and IFVs, Xia and Xu[18] define some new operations on the HFEs h, h_1 and h_2:

$h^{\lambda}=\bigcup_{\gamma\in h}\left\{\gamma^{\lambda}\right\}$;

$\lambda h=\bigcup_{\gamma\in h}\left\{1-(1-\gamma)^{\lambda}\right\}$;

(3) $h_1\oplus h_2=\bigcup_{\gamma_1\in h_1,\gamma_2\in h_2}\left\{\gamma_1+\gamma_2-\gamma_1\gamma_2\right\}$; (4) $h_1\otimes h_2=\bigcup_{\gamma_1\in h_1,\gamma_2\in h_2}\left\{\gamma_1\gamma_2\right\}$.

For real decision making problems, there is always some degree of inter-dependent characteristics between attributes. Usually, there is interaction among attributes of decision makers. However, this assumption is too strong to match decision behaviors in the real world. The independence axiom generally can't be satisfied. Thus, it is necessary to consider this issue.

Definition 3[19]. Let f be a positive real-valued function on X, and μ be a fuzzy measure on X.

The discrete Choquet integral of f with respective to μ is defined by

$$C_{\mu}(f)=\sum_{i=1}^{n} f_{\sigma(i)}\left[\mu\left(A_{\sigma(i)}\right)-\mu\left(A_{\sigma(i-1)}\right)\right] \tag{2}$$

where $(\sigma(1),\sigma(2),\cdots,\sigma(n))$ is a permutation of $(1,2,\cdots,n)$, such that $f_{\sigma(i-1)} \geq f_{\sigma(i)}$ for all $j=2,\cdots,n$, $A_{\sigma(k)}=\left\{x_{\sigma(j)} \mid j \leq k\right\}$, for $k \geq 1$, and $A_{\sigma(0)}=\phi$.

Furthermore, Torra[17] proposed an aggregation principle for HFEs:

Definition **4.** Let $E=\{h_1,h_2,\cdots,h_n\}$ be a set of n HFEs, Θ be a function on E, $\Theta:[0,1]^N \rightarrow [0,1]$, then

$$\Theta_E=\bigcup_{\gamma \in \{h_1 \times h_2 \times \cdots \times h_n\}}\{\Theta(\gamma)\} \tag{3}$$

Based on the aggregation principle for HFEs, Xia and Xu[18] developed the hesitant fuzzy ordered weighted averaging (HFOWA) operator.

Definition **5**[18]**.** Let $h_j\ (j=1,2,\cdots,n)$ be a collection of HFEs, the hesitant fuzzy ordered weighted averaging (HFOWA) operator of dimension n is a mapping HFOWA: $H^n \rightarrow H$, that has an associated weight vector $w=(w_1,w_2,\cdots,w_n)^T$ such that $w_j>0$ and $\sum_{j=1}^{n} w_j=1$. Furthermore,

$$\begin{aligned} HFOWA(h_1,h_2,\cdots,h_n) &= \bigoplus_{j=1}^{n}\left(w_j h_{\sigma(j)}\right) \\ &= \bigcup_{\gamma_{\sigma(1)} \in h_{\sigma(1)},\gamma_{\sigma(2)} \in h_{\sigma(2)},\cdots,\gamma_{\sigma(n)} \in h_{\sigma(n)}}\left\{1-\prod_{j=1}^{n}\left(1-\gamma_{\sigma(j)}\right)^{w_j}\right\} \end{aligned} \tag{4}$$

where $(\sigma(1),\sigma(2),\cdots,\sigma(n))$ is a permutation of $(1,2,\cdots,n)$, such that $h_{\sigma(j-1)} \geq h_{\sigma(j)}$ for all $j=2,\cdots,n$.

Based on Definition 3, In what follows, Wei et al.[20] developed the hesitant fuzzy choquet ordered averaging (HFCOA) operator based on the well-known Choquet integral[19].

Definition **6.** Let Let $h_j\ (j=1,2,\cdots,n)$ be a collection of HFEs on X, and μ be a fuzzy measure on X, then we call

$$\mathrm{HFCOA}_{\mu}(h_1,h_2,\cdots,h_n)=\bigoplus_{j=1}^{n}\left(\left(\mu\left(A_{\sigma(j)}\right)-\mu\left(A_{\sigma(j-1)}\right)\right)h_{\sigma(j)}\right) \tag{5}$$

the hesitant fuzzy choquet ordered averaging (HFCOA) operator, where $(\sigma(1),\sigma(2),\cdots,\sigma(n))$ is a permutation of $(1,2,\cdots,n)$, such that $h_{\sigma(j-1)} \geq h_{\sigma(j)}$ for all $j=2,\cdots,n$, $A_{\sigma(k)}=\left\{x_{\sigma(j)} \mid j \leq k\right\}$, for $k \geq 1$, and $A_{\sigma(0)}=\phi$.

With the operation of hesitant fuzzy elements, the HFCOA operator can be transformed into the following from by induction on n:

$$\begin{aligned} \mathrm{HFCOA}_{\mu}(h_1,h_2,\cdots,h_n) &= \bigoplus_{j=1}^{n}\left(\left(\mu\left(A_{\sigma(j)}\right)-\mu\left(A_{\sigma(j-1)}\right)\right)h_{\sigma(j)}\right) \\ &= \bigcup_{\gamma_{\sigma(1)} \in h_{\sigma(1)},\gamma_{\sigma(2)} \in h_{\sigma(2)},\cdots,\gamma_{\sigma(n)} \in h_{\sigma(n)}}\left\{1-\prod_{j=1}^{n}\left(1-\gamma_{\sigma(j)}\right)^{\mu\left(A_{\sigma(j)}\right)-\mu\left(A_{\sigma(j-1)}\right)}\right\} \end{aligned} \tag{6}$$

whose aggregated value is also a hesitant fuzzy elements.

3. An Approach to Evaluating the Three-dimension Reconstruction Image Quality with Hesitant Fuzzy Information

The problem of evaluating the three-dimension reconstruction image quality with hesitant fuzzy information is the multiple attribute decision making (MADM) problems. The aim of this section is to investigate the MADM problems for evaluating the three-dimension reconstruction image quality with hesitant fuzzy information. Let $A=\{A_1,A_2,\cdots,A_m\}$ be a discrete set of alternatives, and $G=\{G_1,G_2,\cdots,G_n\}$ be the set of attributes, $\omega=(\omega_1,\omega_2,\cdots,\omega_n)$ is the weighting vector of the attribute $G_j\ (j=1,2,\cdots,n)$, where $\omega_j\in[0,1]$, $\sum_{j=1}^{n}\omega_j=1$. If the decision makers provide several values for the alternative A_i under the attribute G_j with anonymity, these values can be considered as a hesitant fuzzy element h_{ij}. In the case where two decision makers provide the same value, then the value emerges only once in h_{ij}. Suppose that the decision matrix $H=(h_{ij})_{m\times n}$ is the hesitant fuzzy decision matrix, where $h_{ij}\ (i=1,2,\cdots,m,\ j=1,2,\cdots,n)$ are in the form of HFEs.

In the following, we apply the hesitant fuzzy choquet ordered averaging (HFCOA) operator to multiple attribute decision making (MADM) problems for evaluating the three-dimension reconstruction image quality with hesitant fuzzy information.

Step 1. We utilize the decision information given in matrix R, and the HFCOA operator

$$\begin{aligned} h_i &= HFCOA(h_{i1},h_{i2},\cdots,h_{in}) = \oplus_{j=1}^{n}\left(\left(\mu\left(A_{\sigma(j)}\right)-\mu\left(A_{\sigma(j-1)}\right)\right)h_{\sigma(j)}\right) \\ &= \cup_{\gamma_{\sigma(1)}\in h_{\sigma(1)},\gamma_{\sigma(2)}\in h_{\sigma(2)},\cdots,\gamma_{\sigma(n)}\in h_{\sigma(n)}}\left\{1-\prod_{j=1}^{n}\left(1-\gamma_{\sigma(j)}\right)^{\mu\left(A_{\sigma(j)}\right)-\mu\left(A_{\sigma(j-1)}\right)}\right\} \end{aligned},\ i=1,2,\cdots,m. \quad (7)$$

to derive the overall preference values $h_i\ (i=1,2,\cdots,m)$ of the alternative A_i.

Step 2. Calculate the scores $S(h_i)\ (i=1,2,\cdots,m)$ of the overall hesitant fuzzy preference values $h_i\ (i=1,2,\cdots,m)$ to rank all the alternatives $A_i\ (i=1,2,\cdots,m)$ and then to select the best one(s).

Step 3. Rank all the alternatives $A_i\ (i=1,2,\cdots,m)$ and select the best one(s) in accordance with $S(h_i)\ (i=1,2,\cdots,m)$.

Step 4. End.

4. Illustrative Example

Reconstructing the three-dimensional (3D) structure of a scene from two-dimensional (2D) images is a fundamental problem in computer vision. After decades' of extensive study of image-based 3D reconstruction, this topic remains quite active as evidenced by continued rapid progress being made in the last decade. The task of image-based3D re-construction is the reverse process of image capturing, which corresponds to estimating all (or some) camera parameters and 3D locations of the scene points from their2D observations. This problem poses many challenges due to unavoidable noise and outliers in the data. In this section, we present an empirical case study of evaluating the three-dimension reconstruction image quality with hesitant fuzzy information. Let us suppose there is an image company, which wants to invest a sum of money in the best option. There is a panel with five possible three-dimension reconstruction image companies to invest the money: The image investment company must take a decision according to the following four attributes: ①G_1 is the functionality and reliability; ②G_2 is the efficiency; ③G_3 is the easiness to use; ④G_4 is the maintainability and transferability. In order to avoid influence each other, the decision makers are required to evaluate the five possible markets $A_i\left(i=1,2,\cdots,5\right)$ under the above four attributes in anonymity and the decision matrix $H=\left(h_{ij}\right)_{m\times n}$ is presented in Table 1, where $h_{ij}\left(i=1,2,3,4,5,\ j=1,2,3,4\right)$ are in the form of HFEs.

Table 1. Hesitant fuzzy decision matrix

	G_1	G_2	G_3	G_4
A_1	(0.2,0.4)	(0.3,0.5,0.7)	(0.7,0.9)	(0.2,0.3,0.5)
A_2	(0.4,0.5)	(0.4,0.6)	(0.6,0.7)	(0.6,0.7)
A_3	(0.7)	(0.4,0.5)	(0.7, 0.8,0.9)	(0.5,0.8)
A_4	(0.5,0.7)	(0.4,0.6)	(0.7,0.8)	(0.2, 0.7)
A_5	(0.3,0.4,0.6)	(0.6,0.8)	(0.7,0.8)	(0.2,0.5)

Then, we utilize the approach developed to get the most desirable three-dimension reconstruction image company.

Step 1. Suppose the fuzzy measure of attribute of $G_j\left(j=1,2,\cdots,n\right)$ and attribute sets of G as follows:

$$\mu\left(G_1\right)=0.35,\mu\left(G_2\right)=0.45,\mu\left(G_3\right)=0.40,\mu\left(G_4\right)=0.20,\mu\left(G_1,G_2\right)=0.60,$$
$$\mu\left(G_1,G_3\right)=0.65,\mu\left(G_1,G_4\right)=0.65,\mu\left(G_2,G_3\right)=0.60,\mu\left(G_2,G_4\right)=0.55,$$
$$\mu\left(G_3,G_4\right)=0.50,\mu\left(G_1,G_2,G_3\right)=0.70,\mu\left(G_1,G_2,G_4\right)=0.80,$$
$$\mu\left(G_1,G_3,G_4\right)=0.80,\mu\left(G_2,G_3,G_4\right)=0.70,\mu\left(G_1,G_2,G_3,G_4\right)=1.00$$

Step 2. We utilize the decision information given in matrix H , and the HFCOA operator to obtain the overall preference values h_i of the three-dimension reconstruction image companies $A_i\left(i=1,2,3,4,5\right)$. Take market A_1 for an example, we have

$$h_1=HFCOA\left(h_{11},h_{12},h_{13},h_{14}\right)=HFCOA\left\{\left(0.2,0.4\right),\left(0.3,0.5,0.7\right),\left(0.7,0.9\right),\left(0.2,0.3,0.5\right)\right\}$$
$$=\mathop{\oplus}_{j=1}^{n}\left(\left(\mu\left(A_{\sigma(j)}\right)-\mu\left(A_{\sigma(j-1)}\right)\right)h_{\sigma(j)}\right)$$
$$=\bigcup\nolimits_{\gamma_{\sigma(11)}\in h_{\sigma(11)},\gamma_{\sigma(12)}\in h_{\sigma(12)},\gamma_{\sigma(13)}\in h_{\sigma(13)},\gamma_{\sigma(14)}\in h_{\sigma(14)}}\left\{1-\prod_{j=1}^{4}\left(1-\gamma_{\sigma(1j)}\right)^{\mu\left(A_{\sigma(j)}\right)-\mu\left(A_{\sigma(j-1)}\right)}\right\}$$
$$=\bigcup\nolimits_{\gamma_{\sigma(11)}\in h_{\sigma(11)},\gamma_{\sigma(12)}\in h_{\sigma(12)},\gamma_{\sigma(13)}\in h_{\sigma(13)},\gamma_{\sigma(14)}\in h_{\sigma(14)}}\left\{1-\left(1-\gamma_{\sigma(11)}\right)^{0.4}\left(1-\gamma_{\sigma(12)}\right)^{0.2}\left(1-\gamma_{\sigma(13)}\right)^{0.1}\left(1-\gamma_{\sigma(14)}\right)^{0.3}\right\}$$
$$=\{0.4739,0.4808,0.4980,0.5081,0.5146,0.5174,0.5238,0.5307,0.5395,0.5488,0.5548,0.5559,$$
$$0.5618,0.5695,0.5763,0.5926,0.5980,0.6113,0.6610,0.6655,0.6765,0.6830,0.6872,0.6890,$$
$$0.6931,0.6976,0.7033,0.7092,0.7131,0.7138,0.7176,0.7226,0.7269,0.7375,0.7410,0.7495\}$$

Step 3 Calculate the scores $S\left(h_i\right)$ of the overall hesitant fuzzy preference values $h_i\left(i=1,2,3,4\right)$

$$S\left(h_1\right)=0.6234,S\left(h_2\right)=0.5507,S\left(h_3\right)=0.7514$$
$$S\left(h_4\right)=0.5843,S\left(h_5\right)=0.3256$$

Step 4 Rank all the three-dimension reconstruction image companies $A_i\left(i=1,2,3,4,5\right)$ in accordance with the scores $S\left(h_i\right)\left(i=1,2,3,4,5\right)$ of the overall hesitant fuzzy preference values $h_i\left(i=1,2,3,4,5\right)$:

$A_3 \succ A_1 \succ A_4 \succ A_2 \succ A_5$, and thus the most desirable three-dimension reconstruction image company is A_3.

5. Conclusion

Three-dimensional information obtained from two-dimensional images is one of the hot current studies and belongs to a multi-interdisciplinary area of research, which involves the projection geometry, digital image processing, computer graphics, computer vision and many other disciplines. Three-dimensional reconstruction is to restore the three-dimensional space information of objects through the basic elements (such as point, line, plane) of two-dimensional images, and it needs to study the relations between three-dimensional coordinates of points, lines and planes in three-dimensional space and the corresponding ones in two-dimensional images, for achieving quantitative analysis of the sizes and positions of objects. The three-dimensional information or three-dimensional model, which is obtained by features extraction, features matching, reconstruction of key characteristics, triangulation, and data fusion, is widely used in many fields like visualization of the virtual plant, digital entertainment, appearance design of industrial products and virtual scene simulation. Reconstruction for three-dimensional objects based on two-dimensional images has made some achievements. The existing reconstruction methods can produce acceptable results for the regular profile curves (such as hyperbolic, parabolic, etc.) or man-made object profiles (housing, furniture, etc.). However, for the nature scenery or irregular profile reconstruction, the existing methods have many limitations. The problem of evaluating the three-dimension reconstruction image quality with hesitant fuzzy information is the multiple attribute decision making (MADM) problems. In this paper, we investigate the multiple attribute decision making (MADM) problems for evaluating the three-dimension reconstruction image quality with hesitant fuzzy information. We utilize the hesitant fuzzy choquet ordered averaging (HFCOA) operator to aggregate the hesitant fuzzy information corresponding to each alternative and get the overall value of the three-dimension reconstruction image quality, then rank the venture capital project and select the most desirable one(s) by using the score functions of hesitant fuzzy values. Finally, an illustrative example for evaluating the three-dimension reconstruction image quality is given to verify the developed approach and to demonstrate its practicality and effectiveness.

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