Exact Outage Probability of Spectrum-Sharing DF Relaying with Transmit Antenna Selection

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Abstract

Considered is a spectrum-sharing network consisting of a primary user and a secondary decode-and-forward relaying system with a direct source-destination link, then a transmit antenna selection (TAS) scheme for the secondary relaying system are investigated over Rayleigh fading channels, and an exact closed-form expression for the outage probability are derived. Furthermore, Monte Carlo simulations are presented to confirm the analytical result, and show that the TAS scheme can also improve the diversity gain in the spectrum-sharing network.

Keywords

Spectrum-sharing, Transmit antenna selection, Decode-and-forward Relaying.

1. Introduction

Transmit antenna selection (TAS) schemes have been extensively investigated in two-hop relaying systems [1-4], and it has been shown that these TAS schemes can not only improve the diversity gain but also reduce the system complexity. In addition, the performance analysis for cognitive radio with applications to relaying systems has gained much attention in the research community [5, 6]. But there are few works to evaluate the TAS schemes for relaying systems in the spectrum-sharing scenario.

In this Letter, we investigate the performance of a TAS scheme over Rayleigh fading channels for a secondary decode-and-forward (DF) relaying system which coexists with a primary user (PU) in a spectrum-sharing network. An exact closed-form expression for the outage probability is derived and corroborated by Monte Carlo simulations. Moreover, the analytical result shows that the TAS scheme can achieve more diversity gain with more transmit antennas at the secondary source.

2. System and Channel Model

We consider a spectrum-sharing network where a secondary two-hop relaying system coexists with a primary user (PU). In the secondary two-hop relaying system, a source S employs a relay R to communicate with a destination D using the DF protocol, and the direct link between S and D exists and can not be neglected. We assume that S is equipped with N_{t} antennas and both R and D have a single antenna. The secondary relaying system operates in half-duplex mode, and then the end-to-end information transmission occupies two time slots. During the first time slot, S selects a certain transmit antenna to broadcast its signal to R and D. During the second time slot, if the source information can be fully decoded at R, then R regenerates and transmits the source information to D; otherwise, R remains idle.

Let $S^{(i)}$ denote the ith transmit antenna at S, and the channel coefficients of the links $S^{(i)} \to \mathrm{PU}$, $S^{(i)} \to D$, $S^{(i)} \to R$, $R \to \mathrm{PU}$ and $R \to D$ are denoted by g_i, $h_{\mathrm{SD},i}$, $h_{\mathrm{SR},i}$, f and h_{RD}, respectively. To ensure that the secondary system does not cause any harmful interference on the PU, the transmit power at $S^{(i)}$ and R can be set as $P_{\mathrm{S},i} = I_{\mathrm{p}} / |g_i|^2$ and $P_{\mathrm{R}} = I_{\mathrm{p}} / |f|^2$, where I_{p} denotes the maximum tolerate interference power at the PU [5]. Thus, the instantaneous signal-to-noise ratios (SNRs) of the links $S^{(i)} \to D$, $S^{(i)} \to R$ and $R \to D$ are given by $\gamma_{\mathrm{SD},i} = \gamma_{\mathrm{p}} \left|h_{\mathrm{SD},i}\right|^2 / \left|g_i\right|^2$, $\gamma_{\mathrm{SR},i} = \gamma_{\mathrm{p}} \left|h_{\mathrm{SR},i}\right|^2 / \left|g_i\right|^2$ and $\gamma_{\mathrm{RD}} = \gamma_{\mathrm{p}} \left|h_{\mathrm{RD}}\right|^2 / \left|f\right|^2$, where $\gamma_{\mathrm{p}} \triangleq I_{\mathrm{p}} / N_0$, and N_0 denotes the variance of the additive white Gaussian noise at R and D. It is obvious that these two random variables $\gamma_{\mathrm{SD},i}$ and $\gamma_{\mathrm{SR},i}$ are not statistically independent.

The optimal transmit antenna $S^{(k)}$ at the source can be selected as

$$k = \begin{cases} \arg\max\limits_{1 \le i \le N_t} \gamma_{\mathrm{SD},i}, & |C| = 0 \\ \arg\max\limits_{i \in C} \gamma_{\mathrm{SD},i}, & |C| \ne 0 \end{cases} \tag{1}$$

where $C = \left\{ i \,\middle|\, 1 \le i \le N_t, \gamma_{\mathrm{SR},i} \ge \gamma_{\mathrm{th}} \right\}$ denotes the decoding set, and $|C|$ denotes the cardinality of the decoding set [4]. Thus, the end-to-end instantaneous SNR Γ can be expressed as follows:

$$\Gamma = \begin{cases} \max\limits_{1 \le i \le N_t} \gamma_{\mathrm{SD},i}, & |C| = 0 \\ \max\limits_{i \in C} \gamma_{\mathrm{SD},i} + \gamma_{\mathrm{RD}}, & |C| \ne 0 \end{cases} \tag{2}$$

Furthermore, we assume that all the channels are subject to independent Rayleigh fading, so $\left|g_i\right|^2$, $\left|h_{\mathrm{SD},i}\right|^2$, $\left|h_{\mathrm{SR},i}\right|^2$, $\left|f\right|^2$ and $\left|h_{\mathrm{RD}}\right|^2$ follow independent exponential distribution with mean Ω_{g}, Ω_{SD}, Ω_{SR}, Ω_{f} and Ω_{RD}, respectively. Then the distribution of $\gamma_{\mathrm{SD},i}$, $\gamma_{\mathrm{SR},i}$, and γ_{RD} can be given by the general case as follows: let $Z \triangleq \gamma_{\mathrm{p}} X / Y$, where X and Y follow independent exponential distribution with mean Ω_{x} and Ω_{y}, respectively, then the cumulative distribution function (CDF) and probability density function (PDF) of Z can be given respectively by

$$F_Z(z) = \frac{\lambda z}{\gamma_{\mathrm{p}} + \lambda z}, \quad f_Z(z) = \frac{\lambda \gamma_{\mathrm{p}}}{\left(\gamma_{\mathrm{p}} + \lambda z\right)^2} \tag{3}$$

where $\lambda \triangleq \Omega_{\mathrm{y}} / \Omega_{\mathrm{x}}$. Let $\lambda_{\mathrm{SD}} \triangleq \Omega_{\mathrm{g}} / \Omega_{\mathrm{SD}}$, $\lambda_{\mathrm{SR}} \triangleq \Omega_{\mathrm{g}} / \Omega_{\mathrm{SR}}$ and $\lambda_{\mathrm{RD}} \triangleq \Omega_{\mathrm{f}} / \Omega_{\mathrm{RD}}$, then the CDFs and PDFs of $\gamma_{\mathrm{SD},i}$, $\gamma_{\mathrm{SR},i}$ and γ_{RD} possess the same forms as (3).

3. Outage Probability

In accordance with the law of total probability, the outage probability can be expressed as

$$P_{\mathrm{out}} = \Pr\left\{\Gamma < \gamma_{\mathrm{th}}\right\} = I_0 + \sum_{n=1}^{N_{\mathrm{t}}} \binom{N_{\mathrm{t}}}{n} \Pr\left\{|C| = n\right\} I_n \tag{4}$$

where I_n, $n \in \left\{0, 1, \ldots, N_{\mathrm{t}}\right\}$, denotes the outage probability when the cardinality of the decoding set is n, and

$$\Pr\left\{|C| = n\right\} = \left[1 - F_{\gamma_{\mathrm{SR},i}}\left(\gamma_{\mathrm{th}}\right)\right]^n \left[F_{\gamma_{\mathrm{SR},i}}\left(\gamma_{\mathrm{th}}\right)\right]^{N_{\mathrm{t}} - n} = \left(\frac{\rho}{\rho + \lambda_{\mathrm{SR}}}\right)^n \left(\frac{\lambda_{\mathrm{SR}}}{\rho + \lambda_{\mathrm{SR}}}\right)^{N_{\mathrm{t}} - n} \tag{5}$$

where $F_{\gamma_{\mathrm{SR},i}}(\cdot)$ denotes the CDF of $\gamma_{\mathrm{SR},i}$, γ_{th} denotes a certain threshold SNR, and $\rho \triangleq \gamma_{\mathrm{p}} / \gamma_{\mathrm{th}}$ can be treated as the normalised SNR. Firstly, I_0 can be derived as follows:

$$
\begin{aligned}
I_0 &= \left[\Pr\left\{\gamma_{\mathrm{SD},i} < \gamma_{\mathrm{th}}, \gamma_{\mathrm{SR},i} < \gamma_{\mathrm{th}}\right\}\right]^{N_{\mathrm{t}}} \\
&= \left[\int_0^{\infty} \Pr\left\{\left|h_{\mathrm{SR},i}\right|^2 < \frac{x}{\rho}\right\} \Pr\left\{\left|h_{\mathrm{SD},i}\right|^2 < \frac{x}{\rho}\right\} f_{|g_i|^2}(x)\,dx\right]^{N_{\mathrm{t}}} \\
&= \left[1 + \rho\left(\frac{1}{\lambda_{\mathrm{SR}} + \lambda_{\mathrm{SD}} + \rho} - \frac{1}{\lambda_{\mathrm{SR}} + \rho} - \frac{1}{\lambda_{\mathrm{SD}} + \rho}\right)\right]^{N_{\mathrm{t}}}
\end{aligned} \tag{6}
$$

where $f_{|g_i|^2}(\cdot)$ denotes the PDF of $|g_i|^2$. Moreover, conditioned on $\gamma_{\mathrm{SR},i} > \gamma_{\mathrm{th}}$, we can defined a new random variable as $Z_i \triangleq \gamma_{\mathrm{SD},i} \left| \left(\gamma_{\mathrm{SR},i} > \gamma_{\mathrm{th}}\right)\right.$, then I_n, $n \in \{1, \ldots, N_{\mathrm{t}}\}$, can be expressed as

$$
I_n \triangleq \Pr\left\{\max_{1 \le i \le n} Z_i + \gamma_{\mathrm{RD}} < \gamma_{\mathrm{th}}\right\} \tag{7}
$$

According to Bayes' theorem, the CDF of Z_i can be derived as follows:

$$
F_{Z_i}(z) = \Pr\left\{\gamma_{\mathrm{SD},i} < z \middle| \gamma_{\mathrm{SR},i} > \gamma_{\mathrm{th}}\right\} = \frac{\Pr\left\{\gamma_{\mathrm{SD},i} < z, \gamma_{\mathrm{SR},i} > \gamma_{\mathrm{th}}\right\}}{\Pr\left\{\gamma_{\mathrm{SR},i} > \gamma_{\mathrm{th}}\right\}} = 1 - \frac{\gamma_{\mathrm{p}} + \lambda_{\mathrm{SR}}\gamma_{\mathrm{th}}}{\lambda_{\mathrm{SD}} z + \gamma_{\mathrm{p}} + \lambda_{\mathrm{SR}}\gamma_{\mathrm{th}}} \tag{8}
$$

Then the CDF of $\max_{1 \le i \le n} Z_i$ can be given by $F_{\max_{1 \le i \le n} Z_i}(x) = \left[F_{Z_i}(z)\right]^n$, and I_n, $n \in \{1, \ldots, N_{\mathrm{t}}\}$, can be derived with the aid of the binomial theorem as follows:

$$
\begin{aligned}
I_n &= \int_0^{\gamma_{\mathrm{th}}} F_{\max_{1 \le i \le n} Z_i}(\gamma_{\mathrm{th}} - x)\, f_{\gamma_{\mathrm{RD}}}(x)\,dx \\
&= \frac{\lambda_{\mathrm{RD}}\gamma_{\mathrm{th}}}{\gamma_{\mathrm{p}} + \lambda_{\mathrm{RD}}\gamma_{\mathrm{th}}} + \frac{\gamma_{\mathrm{p}}}{\lambda_{\mathrm{RD}}} \sum_{k=1}^{n} \binom{n}{k} \left(\frac{\gamma_{\mathrm{p}}}{\lambda_{\mathrm{SD}}} + \frac{\lambda_{\mathrm{SR}}\gamma_{\mathrm{th}}}{\lambda_{\mathrm{SD}}}\right)^k \int_0^{\gamma_{\mathrm{th}}} P(x)\,dx
\end{aligned} \tag{9}
$$

where $f_{\gamma_{\mathrm{RD}}}(\cdot)$ denotes the PDF of γ_{RD}, and

$$
P(x) \triangleq \frac{1}{\left(x + \frac{\gamma_{\mathrm{p}}}{\lambda_{\mathrm{RD}}}\right)^2 \left[x - \left(\gamma_{\mathrm{th}} + \frac{\gamma_{\mathrm{p}}}{\lambda_{\mathrm{SD}}} + \frac{\lambda_{\mathrm{SR}}\gamma_{\mathrm{th}}}{\lambda_{\mathrm{SD}}}\right)\right]^k} \tag{10}
$$

Using [7, eq. (2.102)], we can obtain the partial faction expansion of $P(x)$ as follows:

$$
\begin{aligned}
P(x) &= \frac{A_2}{\left(x + \frac{\gamma_{\mathrm{p}}}{\lambda_{\mathrm{RD}}}\right)^2} + \frac{A_1}{x + \frac{\gamma_{\mathrm{p}}}{\lambda_{\mathrm{RD}}}} + \frac{B_1}{x - \left(\gamma_{\mathrm{th}} + \frac{\gamma_{\mathrm{p}} + \lambda_{\mathrm{SR}}\gamma_{\mathrm{th}}}{\lambda_{\mathrm{SD}}}\right)} \\
&\quad + \sum_{j=1}^{k-1} \frac{B_{k-j+1}}{\left[x - \left(\gamma_{\mathrm{th}} + \frac{\gamma_{\mathrm{p}}}{\lambda_{\mathrm{SD}}} + \frac{\lambda_{\mathrm{SR}}\gamma_{\mathrm{th}}}{\lambda_{\mathrm{SD}}}\right)\right]^{k-j+1}}
\end{aligned} \tag{11}
$$

where $A_2 = 1 \big/ \left(-\gamma_{\mathrm{th}}\Phi\right)^k$, $A_1 = (-k) \big/ \left(-\gamma_{\mathrm{th}}\Phi\right)^{k+1}$, $B_{k-j+1} = \left[(-1)^{j-1} j\right] \big/ \left(\gamma_{\mathrm{th}}\Phi\right)^{j+1}$, $j = 1, 2, \cdots, k$, and $\Phi \triangleq 1 + \lambda_{\mathrm{SR}}/\lambda_{\mathrm{SD}} + \rho/\lambda_{\mathrm{SD}} + \rho/\lambda_{\mathrm{RD}}$. Substituting (11) into (9), and after some manipulations, we can have

$$I_n = \frac{\lambda_{\mathrm{RD}}}{\rho+\lambda_{\mathrm{RD}}} + \frac{\rho}{\lambda_{\mathrm{RD}}}\sum_{k=1}^{n}\binom{n}{k}\left(\frac{\rho+\lambda_{\mathrm{SR}}}{\lambda_{\mathrm{SD}}}\right)^k(-1)^k \left\{\frac{\lambda_{\mathrm{RD}}^2}{\Phi^k(\rho+\lambda_{\mathrm{RD}})\rho} + \frac{k}{\Phi^{k+1}}\ln\left[\frac{(\rho+\lambda_{\mathrm{RD}})(\rho+\lambda_{\mathrm{SR}}+\lambda_{\mathrm{SD}})}{\rho(\rho+\lambda_{\mathrm{SR}})}\right] + \sum_{j=1}^{k-1}\frac{j\lambda_{\mathrm{SD}}^{k-j}}{(k-j)\Phi^{j+1}}\left[\frac{1}{(\rho+\lambda_{\mathrm{SR}})^{k-j}} - \frac{1}{(\rho+\lambda_{\mathrm{SR}}+\lambda_{\mathrm{SD}})^{k-j}}\right]\right\} \tag{12}$$

Finally, substituting (5), (6) and (12) into (4), we can obtain a closed-form expression for the outage probability.

4. Simulation Results

To validate the analytical result, computer simulations are performed to evaluate the outage performance. Without loss of generality, we set $\Omega_{\mathrm{g}} = \Omega_{\mathrm{f}} = 2$, $\Omega_{\mathrm{SD}} = 2.5$, $\Omega_{\mathrm{SR}} = 3$ and $\Omega_{\mathrm{RD}} = 3.5$. Fig. 1 shows the outage probability against ρ for $N_{\mathrm{t}} = 1, 2, 3, 4$. As can be seen from Fig. 1, the simulated result math completely with the analytical result (?), the secondary relaying system with TAS ($N_{\mathrm{t}} \geq 2$) outperform significantly that one without TAS ($N_{\mathrm{t}} = 1$), and the TAS scheme can achieve more diversity gain with larger N_{t}.

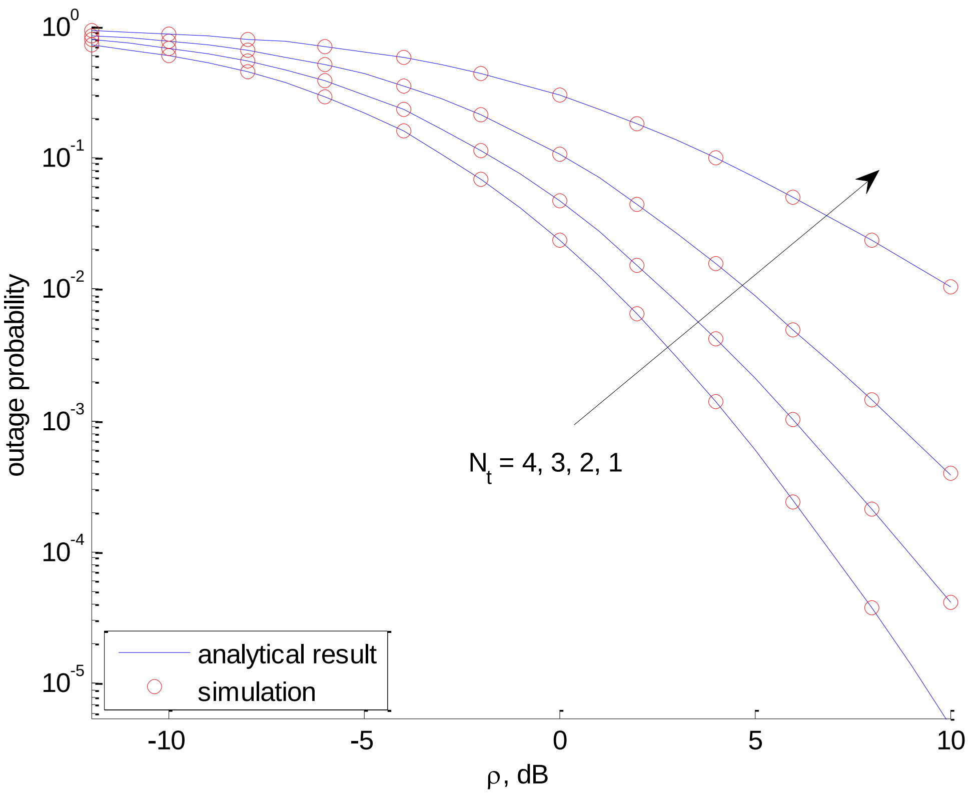

Fig. 1 Outage probability against ρ for $N_{\mathrm{t}} = 1, 2, 3, 4$

5. Conclusion

A TAS scheme for a secondary DF relaying system coexisting with a PU is investigated in a spectrum-sharing network, and an exact closed-form expression for the outage probability is derived. The analytical result shows that the TAS scheme can achieve more diversity gain with more transmit antennas at the secondary source.

Acknowledgements

This work is supported by the Program of HPU Creative Science Researching Team (T2017-2).

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