Research of European Stock Index option pricing of The CSI300 Index based on EGARCH Model

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Abstract

Researches pricing strategies of the CSI 300 stock index options which is still in the stage of simulation trading. Adopts the expansion Black-Scholes option pricing model with the dividend payments, based on the characteristics of financial data, calculates the price of European stock index options under different volatility by historical volatility method and EGARCH model, and then compares the price with simulated transaction data. It provides effective data support for China to issue The CSI300 Index options as soon as possible and to launch various new options. The conclusion can provides reference for the implementation of the policy.

Keywords

The CSI 300 stock index options；Black - Scholes option pricing model；European option；volatility.

1. Introduction

In recent years, with the continuous construction and improvement of China's financial market, new financial derivatives have been launched constantly, and the government has steadily developed the investment and financing market under the premise of controlling the overall financial risks. We will actively give full play to the role of financial support for the development of the real economy. The institutional environment of our country has changed from strict financial regulation to encouraging the marketization and liberalization of innovation, which has greatly promoted the development of financial derivatives. The main function of financial derivatives is risk management. Therefore, whether investors or financiers, the demand for financial derivatives is gradually increasing and option as a derivative product is different from other financial products. It makes the buyer lose limited profit and the seller has limited profit and unlimited loss, so the option has a greater application in risk management for investors. China began to run the simulated trading of The CSI300 options in November 2013, which indicates that the option trading project will be officially launched in China at an appropriate time. Of course, options can not be free, which promotes the study of option pricing.

At present, many scholars have made use of the domestic resources to study the option pricing, but because the stock index option of The CSI 300 index is still in the simulation stage, the pricing of this kind of option is not much. Among them, Zhang Yuan Kun and Yang Hua [1] based on B-S option pricing model to carry on the simulation empirical research to The CSI 300 option pricing, has proved this model to The CSI 300 option pricing accuracy, the pricing is quite effective. Chen ran [2] selected stochastic volatility option pricing model to analyze the volatility structure of The CSI 300 . It is considered that SVT3 model can be used to price The CSI 300 option reliably, and it is necessary to introduce derivative products of stock index option type in the market.

In the pricing of financial derivatives, option pricing is undoubtedly the most complex, but it is also the key to the sound and orderly operation of the option market. As early as 1973, Fischer Black and Myron Scholes put forward the famous B-S option pricing formula. However, due to the limitation of the application condition of the formula, it is not widely used in practice. In addition, Hull and white [4] use stochastic volatility model and dual Monte-Carlo simulation method to compare the accuracy of approximate display solution, and study and analyze the influence of volatility parameter change on option price. The binary tree method proposed by Ross et al, which subdivides the time period into smaller units, is suitable for more complex option pricing [5]. Therefore, combined with the theoretical facts and data, this paper adopts the extended option pricing model considering dividend payment, taking The CSI 300 as the underlying asset of the option contract, and studies the influence of volatility on the option price, with a view to practical use of more extensive application.

2. Model setting of option pricing of The CSI 300 stock indexes

2.1 Classical Black-Scholes model

Because The CSI 300 stock index option in China is a European option, the European stock index option pricing model is used to study it. In the European stock option pricing, the most classic is the B-S model. The key of using the model is to assume that all investors are risk neutral to the option pricing. There are five basic hypotheses for the classical B-S model. Investors cannot get higher interest rates than other investors through arbitrage; Taking the risk-free interest rate r as the expected return rate of the stock index r is a constant; The price change of the underlying index follows the geometric Brownian motion; Investors can not receive dividend returns during the period of validity of stock index options; There is no short-selling limit in the stock market, investors can make short trading; The stock market is frictionless and all securities are completely divisible.

Based on the above assumptions to derive the pricing formula for European call options, the underlying asset refers to the stock price index.

According to the assumptions, the stochastic process of the underlying asset is:

$$dS = \mu S dt + \sigma S dW$$

Where S denotes the price of the underlying asset, μ denotes the average value of the underlying asset price, and σ denotes the volatility of the underlying asset price, i.e. the standard deviation W denotes the standard Brownian motion with a mean value of 0 and a variance of t.

Suppose f is a derivative price based on S, and according to the famous Ito formula, we have:

$$df = \left(\frac{\partial f}{\partial S}\mu S + \frac{\partial f}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} \right) dt + \frac{\partial f}{\partial S}\sigma S dW$$

To construct a portfolio Φ, in which a derivative is sold and an underlying asset $\frac{\partial f}{\partial S}$ is bought, the value of the entire portfolio is:

$$\Phi = -f + \frac{\partial f}{\partial S} S$$

The value of the portfolio becomes:

$$d\Phi = -df + \frac{\partial f}{\partial S} dS = -\left(\frac{\partial f}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} \right) dt$$

The change in value of the portfolio Φ is only time-dependent dt, and the portfolio eliminates the uncertainty dW that, based on the risk-free arbitrage principle, the portfolio's rate of return should be equal to the risk-free rate of return r:

$$d\Phi = r\Phi dt - \left(\frac{\partial f}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} \right) dt = r\left(-f + \frac{\partial f}{\partial S} S \right) dt$$

Finally, the following differential equations are obtained:

$$\frac{\partial f}{\partial t} + rS\frac{\partial f}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} = rf \qquad (2\text{-}1)$$

The derivative price f of underlying asset S satisfies the above differential equation(2-1). For this differential equation plus the boundary conditions of European call options,

$$C(0,t)=0$$

$$C(S,T)=MAX(S-K,0)$$

The famous B-S European call option pricing formula is obtained by solving the differential equation(2-1):

$$c=SN(d_1)-Ke^{-rT}N(d_2) \quad (2\text{-}2)$$

Where,

$$d_1=\frac{\ln\left(\frac{S}{K}\right)+\left(r-\frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}$$

$$d_2=\frac{\ln\left(\frac{S}{K}\right)+\left(r-\frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}=d_1-\sigma\sqrt{T}$$

Where c represents the price of the European call option, and the executive price is K; Options are valid for T; σ represents the annual fluctuation rate of the underlying asset price , that is , the standard deviation of the annual yield of the underlying asset ; N(X) is the cumulative probability distribution function of the standard normal distribution variable, and according to the characteristic of the standard normal distribution function, $N(-X)=1-N(X)$.

According to the theorem of European option parity formula, on the basis of Black-Scholes option pricing model, it is easy to obtain the European put option pricing formula:

$$p=Ke^{-rT}N(-d_2)-SN(-d_1) \quad (2\text{-}3)$$

2.2 Extended Black-Scholes model

Although the classical B-S pricing formula is relatively simple, it is not widely used in practice, because the third basic assumption condition is often not satisfied and there is usually a certain dividend payment during the contract period of stock index option. The following discussion is to extend the B-S model to the form of a stock payment continuous dividend.

There is a known discontinuous dividend: if a stock pays a known dividend $D(t)$ at a certain time during the period of validity of an option, the present value of the dividend is subtracted from the present value of the stock S, and the adjusted value of the stock S'is substituted into the B-S model: S'=$\left(S-D(t)e^{-rt}\right)$. If there are other dividends during the period of validity, subtract them according to this method. so that the B - S model is deformed to obtain a new formula :

$$c=\left(S-D(t)e^{-rt}\right)N(d_1)-Ke^{-rT}N(d_2) \quad (2\text{-}4)$$

$$p=Ke^{-rT}N(-d_2)-\left(S-D(t)e^{-rt}\right)N(-d_1) \quad (2\text{-}5)$$

The existence of a known continuous dividend: when the return on the stock is a fixed rate of return q based on a continuous compound interest, We can find out the price of the European call option and put option that pay the continuous compound rate of return as long as we replace the Se^{-qT} for S in the form of (2-2) and (2-3),and we can find out the price of the European call option and the put option which pay the continuous compound rate of return securities.

$$c=Se^{-qT}N(d_1)-Ke^{-rT}N(d_2) \quad (2\text{-}6)$$

$$p=Ke^{-rT}N(-d_2)-Se^{-qT}N(-d_1) \quad (2\text{-}7)$$

Where, $d_1 = \dfrac{\ln\left(\dfrac{S}{K}\right) + \left(r - q + \dfrac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}$

$$d_2 = \frac{\ln\left(\dfrac{S}{K}\right) + \left(r - q - \dfrac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}$$

3. Empirical analysis and test

3.1 Selection of variables and data description

The CSI 300 is the underlying asset. In order to carry out the option simulation pricing, we derive the stock index option pricing formula for dividend payment in the second part.The factors influencing the pricing of options are as follows: the current price of the underlying asset S, the risk-free interest rate r, the dividend rate q, the maturity remaining time T, the underlying asset price volatility σ, and the execution price K. Among the five influencing factors, except volatility data, the other four factors can be determined, and volatility needs to be estimated. The estimated effect of volatility will directly affect the pricing effect.

This paper selects the closing price data of The CSI300 from January 4, 2011 to December 31, 2014 for empirical test. This establishes the sequence {close}, and then constructs its logarithmic return rate sequence {r}, establishes the heteroscedasticity model for the sequence {r}, and studies its earnings volatility. The residual maturity of the options contract will then be studied first .

3.1.1 Residual maturity

Options have time value in the period of existence, and the longer the remaining period, the greater the time value, so the value will generally be greater. This is because the longer the remaining period, the greater the chance that the underlying asset price will change from a virtual value to a real value or a deeper degree of real value, that is, the longer the period of time, the greater the chance of profit. The remaining term of The CSI 300 stock index options is calculated according to the actual maturity date of the options. For example, IO1506C3000, due on June 19, May8, the remaining period is 42 days, and the annualized remaining period is 42 / 252=0.166667.

3.1.2 Volatility estimation

Volatility is a measure of the degree of change in the return on investment in underlying assets. In statistics, it is expressed as standard deviation. In economics, there are three explanations for the source of volatility. The first is the impact of macro factors on a certain industrial sector, namely systematic risk; The second is the impact of certain events on the firm, which is non-systemic risk, and finally the effect of investor sentiment on prices.

In the research of volatility, there are four main categories: one is real volatility, which measures the volatility of options in the lifetime, but the real real volatility can not be accurately calculated because the investment return is a stochastic process. Its estimate can only be obtained by mathematical method. The other is historical volatility, which refers to the volatility of return on underlying assets in the past time, which is calculated on the basis of historical data. In many volatility studies, historical volatility is generally regarded as the estimation of actual volatility. The third is the prediction of volatility, which refers to the results obtained by using statistical method to predict the actual volatility. The last one is implied volatility, which is the true reflection of the volatility in the actual option trading, but the implied volatility can not be obtained directly, but can only be derived from the option price and other factors. In B-S model, all pricing factors other than volatility can be determined, only volatility needs to be estimated. Synthesizing the characteristics of the above four volatility rates, the options products of the overall situation of the underlying The CSI300 stock markets in our country have not yet appeared, so they cannot use the implied volatility rate.In this paper, the historical volatility method and EGARCH model prediction method are used for volatility estimation.

Historical volatility method

Volatility is the standard deviation of a continuous compound rate of return over a certain period of time, usually defined as one year. Denotes $R_t = \ln\left(\frac{S_t}{S_{t-1}}\right) = \ln S_t - \ln S_{t-1}$, $\sigma_t^2 = \frac{1}{N-1}\sum_{t=1}^{N}\left(R_t - \frac{1}{N}\sum_{t=1}^{N} R_t\right)$.

Where, St is the daily closing price of the stock index; N is the number of samples; σ_t representsthe historical volatility of the stock index.

The paper uses EVIEWS software to analyze the return series and draws the changing trend of closing price series and yield sequence of The CSI300 stock index (see figure 3-1 / 3-2). From Fig. 3-1 / 3-2, it can be seen that the closing price of the underlying stock index is not stable, while the return sequence is stable, and the data trend accords with the characteristics of financial data and meets the expectation of establishing the model.

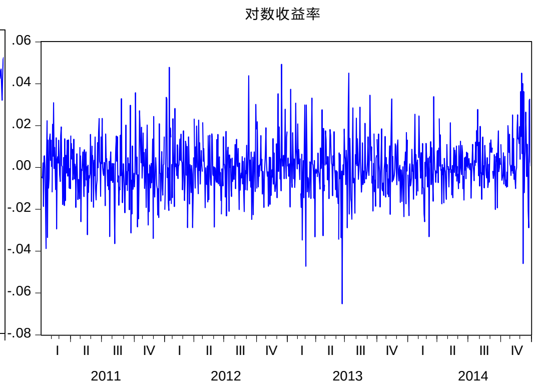

Fig. 3-1 closing price chart of The CSI 300 Fig. 3-2 variation of yield of The CSI 300

A descriptive statistical analysis of the return series is carried out using EVIEWS software. The results are shown in Table 3-1

Table 3-1 Descriptive statistics analysis results

variable name	mean value	standard deviation	J-B statistic	kurtosis	skewness	P value
R_t	0.000106	0.013016	128.3373	4.772199	0.097399	0.0000

As can be seen from Table 3-1, the historical volatility of the underlying stock index is 1.3016, using the formula $\sigma_{year} = \sigma_{day}.\sqrt{T}$,(where: σ_{year} represents annual volatility; σ_{day} represents diurnal volatility ,T is the total number of days traded annually in the stock market), The calculated daily volatility is converted into the corresponding annual volatility, and the annual volatility of The CSI 300 stock index is 20.22306%.

EGARCH model method

Fig. 3-2 shows that the stock price index of The CSI 300 has obvious volatility cluster phenomenon: the large volatility and the smaller volatility are relatively dense, there is a series of volatility aggregation phenomenon, and the existence of ARCH phenomenon in the series is preliminarily judged. To see if there is a real ARCH phenomenon, we'll do some testing next. First, we test whether the return distribution satisfies the normal distribution characteristics.

From Table 3-1, we can see that the average value of logarithmic yield series of The CSI 300 index is 0.000106, and the standard deviation is 0. 013016. The skewness is 0.097399, which is greater than 0, which indicates that the sequence of yield shows the phenomenon of right trailing. The kurtosis value is 4.772199,which is higher than the kurtosis value of normal distribution 3, which indicates that the distribution of logarithmic yield series has obvious phenomenon of sharp peak and thick tail. The Jarque-Bera statistic is 128.3373, p =0.0000000, thus rejecting the assumption that the logarithmic rate of return series is not a normal distribution, that is, the distribution of the logarithmic rate of return is not a normal distribution.

Then we do the ADF test on the smoothness of logarithmic return series:

Null Hypothesis: R has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on SIC, maxlag=16)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-31.14333	0.0000
Test critical values:	1% level	-3.436885	
	5% level	-2.864314	
	10% level	-2.568299	

Fig. 3-3 the results of ADF test on the stationarity of the yield sequence of The CSI 300

From the results in figure 3-3, we can see that the ADF value is -31.14333, and the p value is 0.0000. The absolute values of the absolute values of t statistics are tested under the criteria of 1% or 5% and 10% respectively. Therefore, the return series rejects the original hypothesis at the level of 1% significant, does not have unit root, and the logarithmic return sequence {r} is stable.

Then, we carry on the logarithmic rate of return series autocorrelation, partial autocorrelation test:

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.003	-0.003	0.0111	0.916
		2	0.024	0.024	0.5628	0.755
		3	-0.003	-0.003	0.5716	0.903
		4	0.004	0.003	0.5869	0.965
		5	0.017	0.017	0.8723	0.972
		6	-0.045	-0.045	2.8356	0.829
		7	0.080	0.080	9.1715	0.241
		8	0.028	0.030	9.9166	0.271
		9	0.041	0.037	11.548	0.240
		10	0.059	0.059	14.966	0.133
		11	0.032	0.032	15.982	0.142
		12	-0.044	-0.051	17.863	0.120

Fig. 3-4 autocorrelation and partial correlation of yield series of The CSI 300

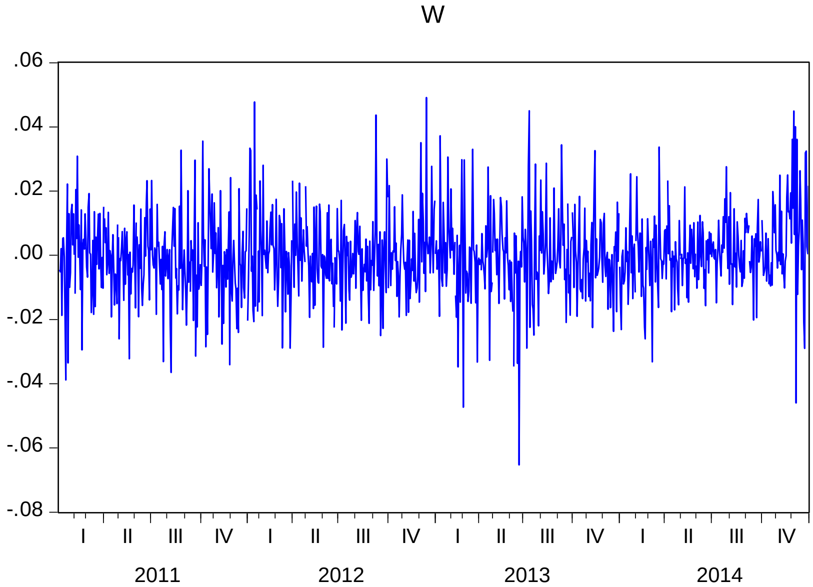

Figure 3-5 return residual diagram of return

From figure 3-4, it can be seen that the autocorrelation and partial correlation coefficients of the sequence fall within the estimated labeling difference of two times, and the p value corresponding to the Q statistic is all greater than 0.05, so there is no significant correlation at the significant level of 5% of the return rate sequence.

Because the sequence has no significant correlation, the mean equation is set as white noise.

Establishing model: $r_t = \pi_t + \varepsilon_t$

By de-meaning r, we get: w=r-0.000106

We further map the regression with residuals:

From figure 3-5, we can see that the residual sequence images also have the phenomenon of wave aggregation.That is, large fluctuations often cause large fluctuations, small fluctuations cause small fluctuations. The residuals are tested by ARCH, as shown in figure 3-6:

Heteroskedasticity Test: ARCH

F-statistic	4.490840	Prob. F(3,962)	0.0039
Obs*R-squared	13.34169	Prob. Chi-Square(3)	0.0040

Test Equation:
Dependent Variable: RESID^2
Method: Least Squares
Date: 04/08/18 Time: 16:31
Sample (adjusted): 1/10/2011 12/31/2014
Included observations: 966 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.000142	1.37E-05	10.39615	0.0000
RESID^2(-1)	0.052348	0.032254	1.623004	0.1049
RESID^2(-2)	0.101024	0.032133	3.143889	0.0017
RESID^2(-3)	0.007781	0.032256	0.241222	0.8094

Fig. 3-6 residual sequence ARCH test results

By F test, the second order lag term of t test is still very significant. It can be seen that there are higher order ARCH effects. The EGARCH model can be built to deal with the time-varying variance characteristics.

According to the criteria of AIC and SC, the model of EGARCH(1,1)、EGARCH(1,2)、EGARCH(2,1) is established to fit the time-varying variance characteristics of the return rate, and the AIC and SC values are obtained as shown in Table 3-2:

Table 3-2 AIC and SC values of each model

model	AIC	SC
EGARCH(1,1)	-5.869959	-5.854365
EGARCH(1,2)	-5.869452	-5.849830
EGARCH(2,1)	-5.869189	-5.849060

According to Table 3-2, the AIC and SC values of EGARCH(1,1) are small, and the effect is the best. EGARCH(1,1) model is established to fit the time-varying variance characteristics. The model is as follows:

$$\ln\left(\sigma^2\right) = -0.387894 + 0.110141\left|\varepsilon_{t-1}\middle/\sqrt{\sigma_{t-1}^{\ 2}}\right| - 0.018790\,\varepsilon_{t-1}\middle/\sqrt{\sigma_{t-1}^{\ 2}} + 0.964710\ln\sigma_{t-1}^{\ 2}$$

$$V_L = \exp\left(\frac{w+\alpha\sqrt{\frac{2}{\pi}}}{1-\beta}+1\right)$$

The EGARCH(1,1) model is used to estimate the volatility of The CSI 300 index. The long-term variance of the return series can be obtained.

$$V_L = \exp(\frac{w+\alpha\sqrt{\frac{2}{\pi}}}{1-\beta}+1) = 0.000302311$$

Corresponding long-term volatility= $0.000302311^{0.5} = 0.023501371$

Because the data we used in the previous model are daily yield data, the long-term volatility is the corresponding daily volatility, and the volatility in the option pricing model we need is annualized volatility. So what is needed here is to annualize the daily volatility and get the annual volatility of The CSI 300 stock index is 33. 30727.

3.1.3 Risk-free rate of interest

Risk-free return is the rate of return that can be obtained by investing funds in a risk-free investment object. In mature capital markets , the yield of a one - year Treasury bond is often used as a risk - free rate of return on investment . The current scale of the national debt market in China is comparable to that of the stock market. According to the China Bond Market Statistics and Analysis report, in 2014, the bond market issued 12.28 trillion RMB of all kinds of bonds, and by the end of December 2014, the total amount of bonds in the national bond market had reached 35.64 trillion RMB. Statistics show that by the end of 2014, market value, 2592 listed companies in China's capital market, had a total size of more than 35 trillion RMB, reaching 37.11 trillion RMB. At the same time, China's interest rate marketization reform has also made great progress, the interest rate marketization process has been further deepened. Therefore, this paper attempts to use the risk-free yield of RMB one-year treasury bonds 2.2735% as the risk-free rate of return on The CSI 300 index option pricing.

3.1.4 The CSI 300 Index dividend yield

Dividend yield, that is, dividend payment rate, is the ratio between the dividend paid by shareholders and the price of the stock. The dividend yield of a single stock is divided by the price of the stock, and the dividend rate of the index shall be calculated according to the respective dividend yield of its constituent stock and the weight of the constituent stock. The calculation of specific exponential dividend yield is relatively cumbersome, and there are related statistical results. The specific index dividend rate is not calculated in detail here. In this paper, the dividend yield of The CSI 300 index in 2014 is chosen as the dividend yield of The CSI 300 stock index option pricing, that is, 2.4589%.

3.2 Empirical analysis and comparison

The volatility is calculated by historical volatility method and EGARCH model, so that all the parameters needed for option pricing have been obtained. As a result, we can calculate the price of European call options with dividends under different volatility rates, and use the parity relationship between call options and put options to find out the price of the corresponding put options (see Table 3-3), Its hanging time is November 20, 2014, due date is December 18, 2014. The underlying index closed at 3345.927 points on December 18, 2014, assuming that the option contract's expiration and execution price K was 3500.00. Compare the result of calculation with the data of the simulated transaction of option simulation, it is found that the EGARCH model method is more accurate and effective than the historical volatility method in terms of option pricing.

Table 3-3 European stock index option pricing results of The CSI 300

project	execution price	Option Price based on Historical volatility method	Option price based on EGARCH (1,1) model
Call option	3500.00	35.55191	163.3828
Put Options	3500.00	189.932	317.763

4. Conclusion

Through the analysis of the pricing results of The CSI300 stock index options, it is shown that although many preconditions are difficult to satisfy, the Black-Scholes option pricing model is still authoritative in option pricing. The solution of volatility is the key of option pricing. EGARCH model method is more accurate and effective than historical volatility method in option pricing.

In recent years, China has vigorously promoted the construction and reform of the financial market, and the variety of financial products is also increasing. While the financial risks are gradually amplified, financial derivatives stand out because of their advantages of price discovery and risk aversion. And option has the uniqueness of asymmetric rights and obligations, so it becomes the preferred financial instrument for investors. Stock index option is the inevitable outcome of capital market development. The introduction of a variety of stock index options helps to improve China's investment and financing market system [6]. Therefore, the implementation and application of The CSI 300 index options is expected in China. The core technology of financial transactions is the correct valuation and pricing of trading financial instruments. Option pricing theory will be more widely used in the future, and its theoretical results are closely related to the actual operation of financial markets and can be used in financial transactions. It can have a huge impact on the practice of financial transactions. The derivatives market is huge and complicated, and many more complex options pricing needs further exploration.

References

[1] Zhang Yuan Kun, Yang Hua. Research on Shanghai and Shenzhen 300 Stock Index option pricing based on Black-Scholes Model [J].Journal of Beihua University (Social Science Edition), 2014,15(02):45-49.

[2]Chen ran. A study on the pricing of Shanghai and Shenzhen 300 Stock Index options-an empirical study based on SV-T Model [J].Journal of Guangxi University (philosophy and Social Sciences Edition): 2015,37(03):71-74.

[3]Black F, Scholes M.The Pricing of Options and Corporate Liabilities[J].Journal of Political Economy,1973,81(3):637-54

[4]Hull,White.The Pricing of Options On Assets with Stochastic Volatilities[J].Journal of Finance, 1987,42(2):281-300

[5]Vipul.Box-spread arbitrage efficiency of Nifty index options:The Indian evidence[J].Journal of Futures Markets,2009,29(6):544-562

[6] Li Yiwei. Empirical study on Shanghai and Shenzhen 300 Stock Index option pricing based on Simulation transaction [D].Zhejiang University of Finance and Economics.