Vibration analysis of axial moving beam based on Galerkin method

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Abstract

In order to study the vibration of the axle driven vehicle bridge, the bridge is equivalent to a simple supported beam model. Based on the Euler Bernoulli beam theory, the nonlinear partial differential equation of the bending deformation of the beam is obtained. The two and eight order truncations are carried out on the Euler beam based on the boundary condition of the simple supported beam and Galerkin truncation theory, and then the MATLAB vibration toolbox is programmed with the dimensionless truncation equation and the related parameters. The bifurcation diagram, time history diagram, phase diagram, Poincare cross section diagram and PSP diagram of moving beam with axial excitation frequency are obtained by simulation. The results show that the axial excitation frequency will affect the transverse vibration state of the moving beam, that is, the higher the truncation times, the more accurate the analysis is.

Keywords

Axle, moving beam, nonlinear vibration, galerkin method, excitation frequency.

1. Introduction

In the practical engineering field, the axial motion beam model is widely used, large to high precision aviation engineering, complex and changeable crossing river bridge and tunnel construction, small to the design of automobile axle, can be considered as axial motion beam in reasonable assumptions.With the help of the MATLAB vibrati toolbox, the nonlinear vibration of Galerkin's two and eight order truncations at both ends of a simple supported Euler beam with axial excitation is studied. The bifurcation diagram of the Euler beam with the changing of the excitation frequency and the time history diagram of the motion response, the phase diagram, the Poincare and the PSP diagram are simulated.on

2. Control Equation of Axial Motion Beam

Based on the Euler-Bernoulli beam theory, the influence of shear force is neglected for the axially moving beam.Reference [3], the length of a simple supported flexible beam at both ends is L. As shown in Figure 1, a rectangular coordinate system is set up. The axial excitation is expressed by P, and w (x, t) represents the displacement of a point on the flexible beam in the direction of y. The partial differential equations derived from the Hamilton principle are as follows:

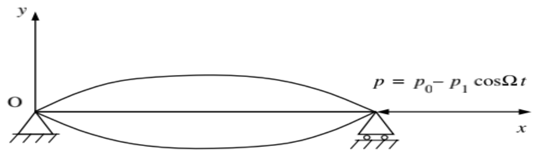

Fig. 1 A model of a simple supported flexible beam at both ends subjected to axial excitation

$$\frac{\partial^2 w}{\partial t^2}+\alpha\frac{\partial w}{\partial t}+\lambda\frac{\partial^4 w}{\partial x^4}+f_1\frac{\partial^2 w}{\partial x^2}-f_2\cos\Omega t\frac{\partial^2 w}{\partial x^2}-\beta\int_0^l\left(\frac{\partial w}{\partial x}\right)^2 dx\frac{\partial^2 w}{\partial x^2}+\delta\int_0^l\left(\frac{\partial w}{\partial x}\right)^4 dx\frac{\partial^2 w}{\partial x^2}=0 \tag{1}$$

Type (1): m is the mass density of the beam, Kg/m; I is the moment of inertia of the cross section of the beam, Kg.m2; E is the Young's modulus of the beam, Pa; C is the damping coefficient of the beam, N/m/s; L is the length of the beam, m; A is the cross section area of the beam. The following dimensionless variables are introduced as follows:

$$w=\bar{w}/l, x=\bar{x}/l, t=\bar{t}\left(EI/ml^4\right)^{1/2}, \Omega=\bar{\Omega}\left(EI/ml^4\right)^{1/2} \tag{2}$$

Among:

$$\alpha=\frac{cl}{\sqrt{mEI}}, \lambda=l^2, \beta=\frac{Al}{2I}, \delta=\frac{Al}{8I}, f_1=\frac{p_0 l^2}{EI}, f_2=\frac{p_1 l^2}{EI} \tag{3}$$

3. Galerkin Method

The flexible beam is a simple beam with two ends, and the boundary condition is:

$$w(0,t)=w''(0,t)=0; w(l,t)=w''(l,t)=0 \tag{4}$$

According to formula (1), the governing equation of an axially moving beam is 5 order nonlinear partial differential equations. It is impossible to obtain the exact solution of the coupling equation of the model. So the Galerkin discretization method is used to transform the nonlinear equation into the ordinary differential equation in the following calculation.The displacement variable of an axially moving beam can be expressed as:

φ(x) is chosen as the characteristic function of the simply supported beam at the ends, and $\varphi_n(x)=\sin(n\pi x)$ is adopted. *N* is the modal number used. $q_n(t)$ is the generalized coordinates of the beam (n=1,2,3,... N). The formula (6) is substituted (3), and then multiplied by the integration of φ_m in the interval 0~1, the N multimode Galerkin truncation can be obtained.among them:

N=2 is taken and the two order Galerkin is truncated.

$$w(x,t)=\sum_{n=1}^{N} q_n(t)\varphi_n(x) \tag{5}$$

$$\int_0^1 \sum_{n,m=1}^{N} \ddot{q}_{n(t)} \sin(n\pi x)\sin m\pi x = \begin{cases} \int_0^1 \sum_{n,m=1}^{N} \ddot{q}_{n(t)} \sin^2(n\pi x)=\frac{1}{2} & m=n \\ \int_0^1 \sum_{n,m=1}^{N} \ddot{q}_{n(t)} \sin(n\pi x)\sin m\pi x=0 & m\neq n \end{cases} = \frac{1}{2}\sum_{n=1}^{N}\ddot{q}_{n(t)} \tag{6}$$

$$\begin{aligned} \int_0^1 [G_N]\sin m\pi x dx = & \sum_{n=1}^{N}\ddot{q}_{n(t)} + \alpha\sum_{n=1}^{N}\dot{q}_{n(t)} + \lambda\sum_{n=1}^{N} q_{n(t)}(n^4\pi^4) - (f_1 - f_2\cos\Omega t)\sum_{n=1}^{N} q_{n(t)}(n^2\pi^2) \\ & + \frac{1}{2}\pi^4\beta\sum_{n=1}^{N} q^2{}_{n(t)} n^2 \sum_{k=1}^{N} q_{k(t)} k^2 - \frac{3}{8}\pi^4\delta\sum_{n=1}^{N} q^4{}_{n(t)} n^4\pi^2 \sum_{k=1}^{N} q_{k(t)} k^2 = 0 \end{aligned} \tag{7}$$

$$\begin{aligned} & \ddot{q}_1+\alpha\dot{q}_1+\lambda\pi^4 q_1-(f_1-f_2\cos\Omega t)\pi^2 q_1+\pi^4\left(\frac{1}{2}\beta q^2{}_1-\frac{3}{8}\delta\pi^2 q^4{}_1\right)(q_1+4q_2)=0 \\ & \ddot{q}_2+\alpha\dot{q}_2+16\lambda\pi^4 q_2-4(f_1-f_2\cos\Omega t)\pi^2 q_2+2\pi^4\left(\beta q^2{}_2-3\delta\pi^2 q^4{}_2\right)(q_1+4q_2)=0 \end{aligned} \tag{8}$$

Then we take *N*=8 and carry out eight order Galerkin truncation.

For the convenience of analysis, the displacement and velocity in the middle of the moving beam are uniformly selected.If *N*=2, displacement is $q=q_1$, velocity is $q`=q`_1$; similarly, *N*=8, displacement is $q=q_1-q_3+q_5-q_7$, velocity is $q`=q`_1-q`_3+q`_5-q`_7$.

4. Numerical Calculation and Analysis

The values of all dimensionless coefficients are given in reference[2]and calculated in the following table:

Table 1 values of dimensionless coefficients

α	f_1	f_2	β	δ	λ
0.8000	9.7176	0.4052	0.0369	0.0005	1.0000

Fig. 2 gives a period doubling bifurcation diagram of displacement and velocity along a dimensionless axial excitation based on Galerkin two order truncated moving beam. Graph (a) is a bifurcation diagram of the axial lateral displacement with the variation of the dimensionless axial excitation frequency. As shown in the figure, it can be seen that with the increase of the axial excitation frequency, the displacement becomes divergent at the beginning, and then slowly begins to focus on a small period of periodic motion near the w=1 and then immediately begins to diverge and then become a periodic motion, at w=1.65, the state of displacement motion changes from double period motion to quasi periodic motion, then bifurcates chaos, and finally concentrates near 0. Graph (b) is a bifurcation diagram of velocity varying with axial excitation frequency. As it can be seen in the picture, the general trend is similar to that of (a), but the variation of velocity with the excitation frequency is significantly smaller than that of the displacement, especially in the range of 0~0.85, so it is also shown that the motion is a nonlinear motion that is accelerated. Generally speaking, the transverse vibration state of a moving beam can be summarized as the law of periodic motion chaotic motion periodic motion.

As shown in Fig. 3 (a) and (b), the bifurcation diagrams of the intermediate displacement and velocity of the moving beam after the eight order truncation of Galerkin are shown to vary with the dimensionless axial excitation frequency. The overall trend is the same as that in Figure 2, but the bifurcation diagram of the velocity with the axial excitation varies greatly. It is obvious that the velocity after the eight order truncation is more dispersed at the point near the 0 point, and the vibration is more complex.

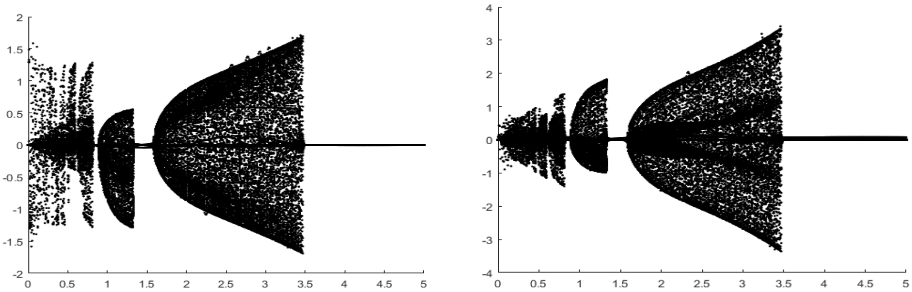

Fig. 2 (a) Displacement

Fig. 2 (b) Velocity

Fig. 2 Bifurcation diagrams of Galerkin's two order truncation along the dimensionless axial excitation frequency.

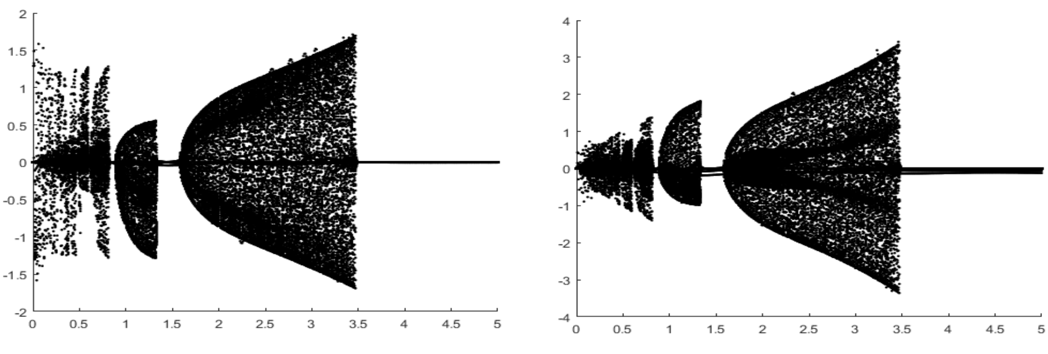

Fig. 3 (a) Displacement

Fig. 3 (b) Velocity

Fig. 3 Bifurcation diagrams of Galerkin's eight order truncation along the dimensionless axial excitation frequency.

Figures 4, 5, 6 and 7, (a), (b), (c), (d), respectively give the time history chart of the *w*=3.5, *w*=1, and *w*=0.5 after two truncation and eight order truncation, the phase diagram, the Poincare cross section and the PSP diagram. The phase diagram is closed and the PSP diagram is a straight line, which represents a periodic motion. The phase diagram is not closed. The PSP diagram is a chaotic point. Therefore, from figure 4 and figure 5, it can be seen that the motion beam is a single periodic motion at this time. In Figure 6, it can be seen that the motion beam is at this time of two times the periodic motion, and the figure 7 can see that the motion beam is chaotic at this time. Because the motion state of a moving beam is certain, the general trend of the graph is the same after a different order of truncation, but it can be seen that there is a difference in the state of chaos by the contrast of Figure 7 (a), (b), (c) and (d). The comparison of time course diagram and phase diagram shows that the trend of vibration is almost the opposite at the same time.

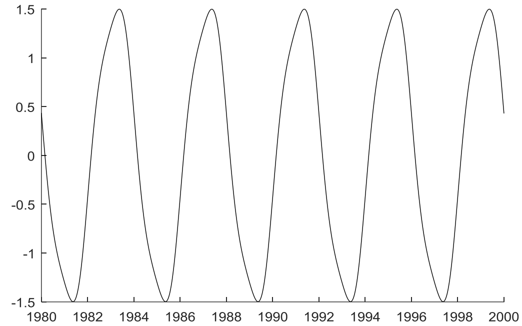

Fig. 4 (a) Time history diagram

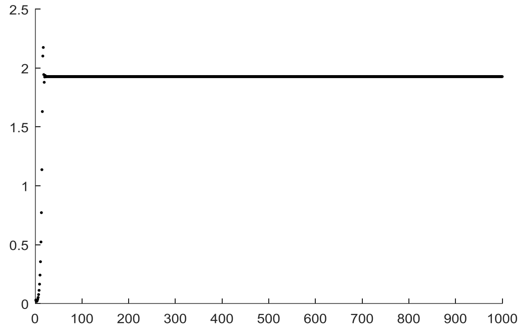

Fig. 4 (b) PSP diagram

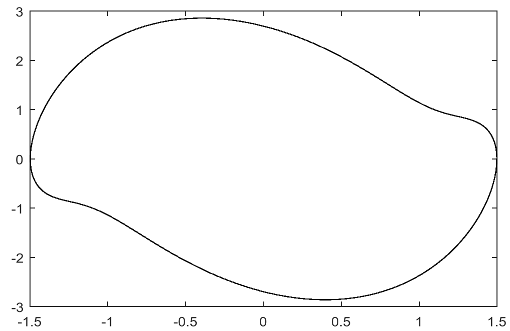

Fig. 4 (c) Phase plane diagram

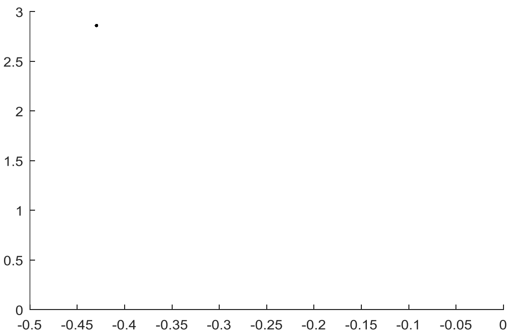

Fig. 4 (d) Poincare diagram

Fig. 4 The time history, phase diagram, Pointcare diagram and PSP diagram of the axially moving beam of w=3.5 after Galerkin's two order truncation.

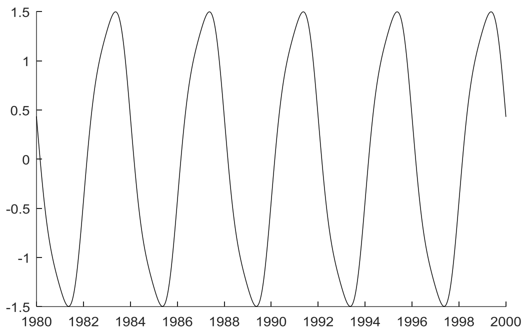

Fig. 5 (a) Time history diagram

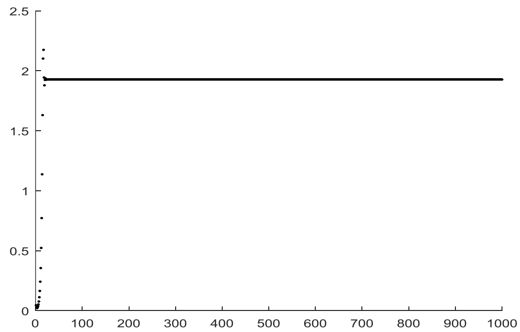

Fig. 5 (b) PSP diagram

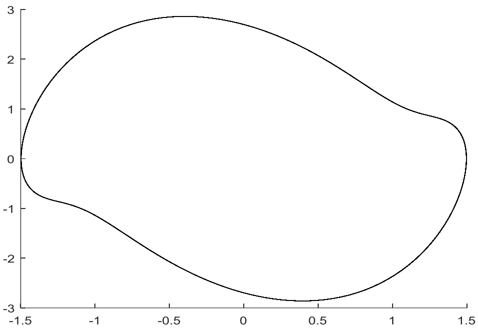

Fig. 5 (c) Phase plane diagram

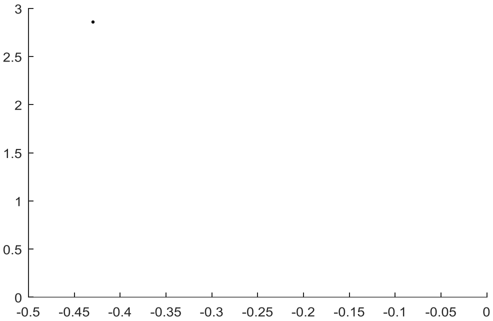

Fig. 5 (d) Poincare diagram

Fig. 5 The time history, phase diagram, Pointcare diagram and PSP diagram of the axially moving beam of w=3.5 after Galerkin's eight order truncation.

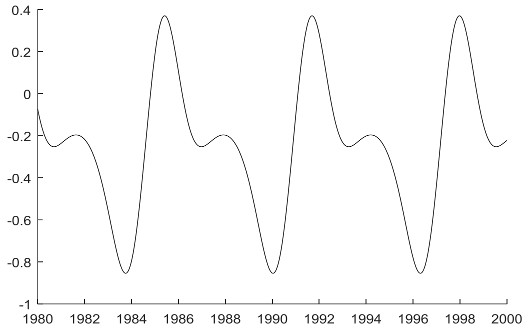

Fig. 6 (a) Time history diagram

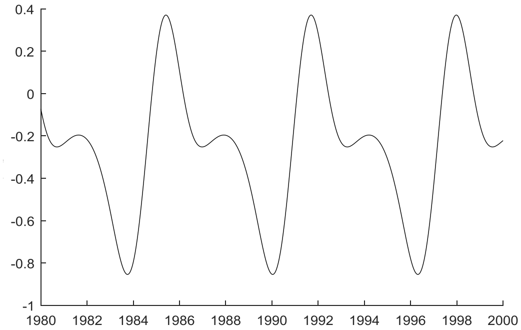

Fig. 6 (b) Time history diagram

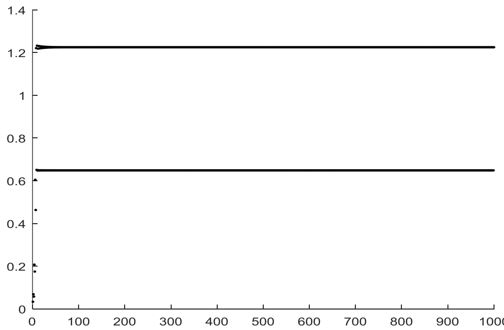

Fig. 6 (a) PSP diagram

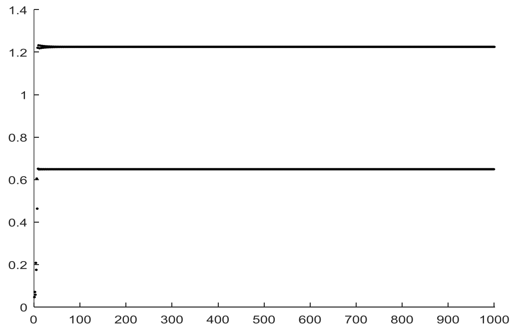

Fig. 6 (b) PSP diagram

Fig. 6 Time histories and PSP diagrams of axially moving beams at two w=1 after Galerkin's two (left) and eight (right) order truncation.

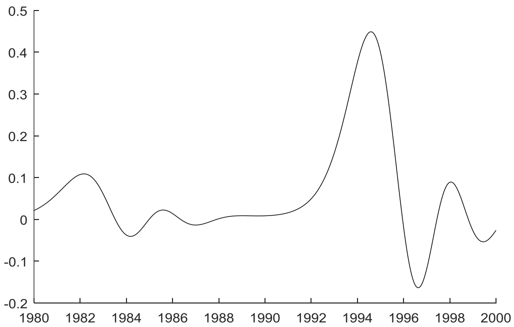

Fig. 7 (a) Time history diagram

Fig. 7 (b) Time history diagram

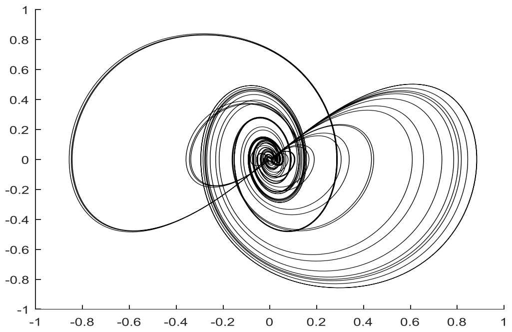

Fig. 7 (a) Phase plane diagram

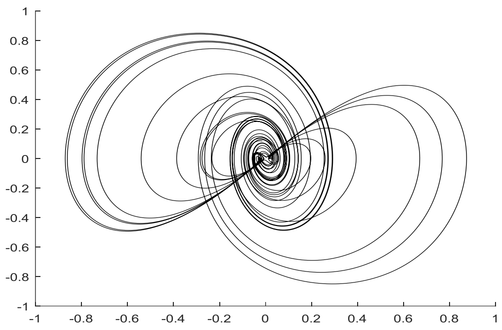

Fig. 7 (b) Phase plane diagram

Fig. 7 The time history and phase diagram of the motion of axially moving w=0.5 after Galerkin's two (left) and eight (right) order truncation.

From the above time history diagram, the phase diagram, the PSP diagram and the ponhal cross section diagram, it can be seen that the bifurcation diagram is consistent with the bifurcation diagram. The vibration of the beam subjected to the axial excitation shows the periodic motion and the chaotic motion.

5. Conclusion

Based on the Galerkin method, the transverse vibration of a simply supported beam subjected to axial excitation is numerically simulated and analyzed through the MATLAB vibration toolbox.

(1). The motion beam is subjected to lateral vibration under the action of axial excitation. With the increase of the excitation frequency, the periodic motion and the chaotic motion occur alternately with the increase of the excitation frequency, and the vibration will be close to the stationary state near the 0 point when the excitation frequency increases to a certain critical value.

(2). By using the Galerkin truncation by using the Runge Kutta method, the nonlinear partial differential equation is transformed into a ordinary differential equation, and the complexity of the operation is greatly reduced by numerical analysis. By comparing the two order truncation and the eight order truncation, it can be seen that the bifurcation diagram obtained with the increase of order is more delicate and closer to the actual situation. Therefore, the accuracy of theory is higher, which is also consistent with the research conclusion of document three.

(3). According to the bifurcation diagram of velocity and displacement with axial excitation frequency, the effect of dimensionless excitation frequency on the lateral vibration velocity and amplitude of the axially moving beam is basically the same.

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