# The Stick-slip Mechanism of Drill String with Axial Vibration

Jialin Tian, Jie Xiong, Ziyi Liu

School of Mechanical Engineering, Southwest Petroleum University, Chengdu, 610500, China

## Abstract

For the current study on the stick-slip of drill string and the influence of axial vibration, the stick-slip mechanism of drill string with axial vibration is researched. Firstly, according to the axial vibration, mechanics analysis model of bit on bottom hole is established. On that basic, mechanics model of drill string stick-slip with axial vibration and the relationship between rotating speed and torque of bit are analyzed. Then the reliability of the calculation method is verified combining with the case analysis by using the built model. The results show that there have close relations between the working parameters of the rotary table drill speed, rotation angular displacement and drill torque. With the rotating speed of table increasing, drill speed and rotation angular displacement and drill torque decreased gradually. The conclusions have some important reference significances on reducing the drill string stick-slip and improving the efficiency of rock-breaking.

## **Keywords**

## Drilling; Drill string; Vibration; Stick-slip; Dynamics.

## **1.** Introduction

With oil and gas exploration deepening, the drilling engineering faces more complex operating conditions, including the process of drilling deep, ultra deep, laterals or directional wells, as well as shale gas or coal-bed gas and so on. In the larger friction condition, with the friction between the drill string and borehole wall increasing, the friction torque in bottom hole increases and the drill string rotates difficultly, which resulting in stick-slip phenomenon[1-3]. When the stick-slip phenomenon of the drill string generates, the drill string will be tighten like clockwork and released suddenly. A serious one can cause huge distortion, which undermines seriously the performance of the drill string and reduces the down-hole tool service life[4-5].

Lots of scientific research has been conducted by scholars at home and abroad to clarify the stick-slip mechanism of drill string[6-10], including the effects to the drill string stick-slip from the drill string length, turntable rotary speed and so on. These studies provide wide ideas to research the stick-slip mechanism of drill string and solve a part of relative problems. However, existing studies only consider the effect of twisting motion and ignore that the stick-slip of drill string is a couple process of axial motion and twisting motion. In fact, during the drilling process, since the bit is under the bottom rock reaction, the drill string will produce axial vibration while the drill string produces torsion vibration, which leads to friction part in the bottom is under a changing constantly state[11-13]. Therefore, the stick-slip only considering torsion vibration can't reflect objective law. The stick-slip of drill string research considering axial vibration reveals essence more efficiently, which is important to reduce stick-slip phenomenon.

## 2. Calculation model

In order to facilitate the calculation, the following assumptions about model are proposed: (1) the quality of drill string system, consisting of the drill string and drill bit, concentrates on the bit; (2) the turntable speed is constant. Then the forces of the drill string are obtained as shown in Figure 1.



Fig. 1 The force on drill string

According to the forces of the drill string, the force equation in the axial direction is written as:

$$M\ddot{x} + K_a x + C_a \dot{x} = F \tag{1}$$

Where the mass matrix is written as:

$$M = \begin{bmatrix} m_1 & & & & \\ & m_2 & & & \\ & & m_3 & & & \\ & & & m_4 & & \\ & & & & & m_n \end{bmatrix}$$
(2)

The stiffness matrix in the axial direction is written as:

The stiffness matrix of each part is written as:

$$k_a = \frac{EA}{L} \tag{4}$$

In which, *x* is the axial displacement of the drill string, *m*; *F* is the axial force of the drill string, *N*; *E* is the elastic modulus of the drill string, GPa; *A* is the cross-sectional area of the drill string,  $m^2$ ; *L* is the length of the drill string, *m*.

According to the forces of the drill bit in the drilling process, the force in the axial direction is written as:

$$F = \begin{bmatrix} 0 \\ 0 \\ 0 \\ . \\ . \\ . \\ . \\ F_N \end{bmatrix}$$
(5)

Where the force at the drill bit is written as:

$$F_N = F_0 + F_{\max} \sin(3\Omega t) - F_b - F_f \tag{6}$$

In which,  $F_0$  is the quiet weight on bit(WOB), N;  $F_{max}$  is the maximum of dynamic WOB, N;  $\Omega$  is the turntable speed, rad/s;  $F_b$  is the impact force on the drill bit resulting in the axial vibration, N;  $F_f$  is the axial friction force of the drill string, N.

The equation in the circumferential direction is written as[14-16]:

$$J\ddot{\theta} + K_t \left(\theta - \Omega t\right) + C_t \dot{\theta} = -T_t \tag{7}$$

The inertia matrix is written as:

$$J = \begin{bmatrix} j_1 & & & & \\ & j_2 & & & \\ & & j_3 & & \\ & & & j_4 & & \\ & & & & \ddots & \\ & & & & & & j_n \end{bmatrix}$$
(8)

Where the torque stiffness is written as:

$$k_t = \frac{GI}{L} \tag{9}$$

The torsional stiffness matrix is written as:

$$K_{t} = k_{t} \begin{bmatrix} 2 & -1 & \cdots & & \\ -1 & 2 & -1 & \cdots & & \\ & -1 & 2 & \cdots & & \\ & & -1 & 2 & & \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ & & & & \cdots & -1 & 2 \end{bmatrix}$$
(10)

In which, J is the inertia of the drill string;  $\theta$  is the rotational displacement of the drill string, rad/s;  $k_t$  is the torsional stiffness of the drill string;  $T_t$  is the total torque of the drill string; G is the shear modulus of the drill string,  $GPa_{\circ}$ 

The torque of the drill string system is written as:

$$T = \begin{bmatrix} 0 \\ 0 \\ 0 \\ . \\ . \\ . \\ . \\ . \\ T_b + T_f \end{bmatrix}$$
(11)

In which,  $T_b$  is the torque at the drill bit,  $N \cdot m$ ;  $T_f$  is the torque of the drill string from the wall,  $N \cdot m$ .

The motion equation of the system is represented by a matrix as:

$$\begin{bmatrix} M & 0 \\ 0 & J \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \ddot{\theta} \end{bmatrix} + \begin{bmatrix} K_a & 0 \\ 0 & K_t \end{bmatrix} \begin{bmatrix} x \\ \theta \end{bmatrix} + \begin{bmatrix} C_a & 0 \\ 0 & C_t \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} F \\ T \end{bmatrix}$$
(12)

When the drill string is under axial vibration, the axial friction force, subjected from the wall, is written as:

$$F_f = \mu M g \tag{13}$$

The torque of the drill string from the wall is written as:

$$T_f = \pi D F_f \tag{14}$$

Assuming that the drill depth is h and the downward movement of the drill string is the positive direction, then

$$h = x \cdot H(t) \tag{15}$$

Where

$$H(t) = \begin{cases} 0 & x \le 0\\ 1 & x > 0 \end{cases}$$
(16)

During the downward vibration of the drill string, according to the relationship of displacement and force, the impact force can be obtained for the drill bit to the bottom[17].

$$F_m = -k_m \left(h\right)^{3/2} \cdot H\left(t\right) \tag{17}$$

Where the equivalent spring stiffness that the drill bit collides with the rock is calculated by the following equation:

$$k_m = 1.9E \cdot \frac{R}{8} \cdot \sqrt{\frac{1}{2}} \tag{18}$$

According to the relationship of the friction force and pressure, the following equation can be obtained:

$$F_{\mu} = fF_N \tag{19}$$

In which,  $F_{\mu}$  the friction force of the drill string at the bottom, N; f is the friction coefficient between the drill bit and the bottom.

Due to the friction force, there is dynamic friction torque while the drill string is rotating during the drilling process. The torque can be calculated by the equation as the following:

$$T_{\mu} = 2\pi R F_{\mu} \tag{20}$$

In which,  $T_{\mu}$  is the dynamic torque of the bit from the bottom,  $N \cdot m$ ; R is the drill bit radius, m.

In the initial moment, turntable rotates at a constant speed. The stick-slip phenomenon of drill string generates, if the torque of the bit that the torsional deformation of the drill string generates is less than the minimum required torque for rock-breaking. And from the initial moment to the drill moment, the drill string satisfies the following equation:

$$k_t \left( \Omega t_m \right) = T_{\min} \tag{21}$$

In which,  $t_m$  is the time from the initial moment to the drill moment, s;  $T_{\min}$  is the minimum required torque for rock breaking,  $N \cdot m$ .

The time from the initial moment to the drill moment can be obtained by the formula (22), and it is written as:

$$t_m = \frac{T_{\min}}{\Omega k_t} \tag{23}$$

In which,  $T_{\min}$  is the minimum required torque for rock breaking,  $N \cdot m$ .

The torque can be calculated by the equation during the process as:

$$T_b = k_t (\theta - \Omega_0 t) \tag{24}$$

The drill bit starts to cut rock with the torque generated by the torsional deformation decreasing. In the process, the rotate of speed (ROP) is increasing from zero to the maximum, then the ROP begins to decrease gradually to zero. During the process, the torque is as the following:

$$T_b = T_\mu \tag{25}$$

The time required is written as:

$$\omega(t = t_p) = \frac{d\theta}{dt} = 0 \tag{26}$$

In which,  $\omega$  is the ROP, rad/s.

Then torsional displacement is calculated by the following equation:

$$\theta_n = \Omega_0 \cdot t_p \tag{27}$$

Assuming that the angular displacement of the drill bit is  $\theta_p$  at the moment, since the torque of the drill bit is less than the minimum rock-breaking torque, the drill string system starts to generate torsional deformation until satisfying the minimum required torque for rock breaking.

In this process, the time required is written as:

$$k_t(\Omega t_k + \theta_n - \theta_p) = T_{\min}$$
(28)

The torque is as the following:

$$T_b = k_t \left( \theta - \Omega_0 t \right) \tag{29}$$

#### 3. Example analysis

According to the calculation method established previously, the example analysis about the stick-slip of the drill string is conducted. On condition that the parameters are given, the calculation content includes the analysis of vibration displacement and vibration velocity. And the example parameters are shown in Table 1.

Table 1 Example parameter

Parameter name	Number Value
The drill string length $L_{,(m)}$	3000
The drill string density $\rho$ , $(kg/m^3)$	7850
The drill string inside diameter $d$ ,( $m$ )	0.12136
The drill string outside diameter $D$ , $(m)$	0.1397
The drill string poisson ratio $\mu$	0.3
The drill string elastic modelling quantity $E$ , (GPa)	210
The drill string shear modulus $G$ , (GPa)	80
The drill bit radius $R$ , $(m)$	0.2
The minimum torque for rock-breaking $T_{\min}$ , $(N \cdot m)$	5400
friction coefficient $f$	0.5
The maximum dynamic WOB $F_{max}$ , (N)	15000
Static WOB $F_0$ ,(N)	30000

According to the given example parameters and established calculation method, the ROP can be obtained in different turntable rotation speed.





From the Fig. 2, considering the stick-slip motion of the drill string under the axial vibration, when the rotary speed of the turntable is constant, since the subjected torque of the drill bit is less than the minimum required torque for rock-breaking, the drill string generates the stick-slip motion in the beginning time. The ROP is zero and the drill string generates torsional deformation until the torque generated by the deformation is more than the minimum torque for rock-breaking. When the stick-slip motion is end, the drill bit begins to generate slipping motion. Since the torque of the drill string generated by deformation is more than the dynamic friction torque of the drill bit is more than the dynamic friction torque of the drill bit is more than the drill sting generated by deformation, the ROP is increasing gradually. When the dynamic friction torque of the drill bit is more than the drill string generates the stick-slip motion once a time. Considering the stick-slip motion of the drill string substituted the axial forces, the period of the stick-slip motion is related to the rotary speed of the turntable, and it presents a decreasing tendency with the rotary speed of the turntable increasing.



Fig. 3 The rotating displacement of drill bit

From the Fig.3, considering the stick-slip motion of the drill string under the axial vibration, when the rotary speed of the turntable is constant, since the ROP is zero, the drill bit remains stationary.,

and the rotational displacement is zero in the beginning moment. When the stick-slip motion is end, the drill bit begins to generate slipping motion. The ROP increases firstly and then decreases, however, the rotational displacement increases all the time, and the rate of change is also increasing firstly and then decreases until the drill bit generates the stick-slip motion. Considering the stick-slip motion of the drill string substituted the axial forces, the period of the rotational displacement is related to the rotary speed of the turntable, and it presents a decreasing tendency with the rotary speed of the turntable increasing.



Fig. 4 The torque on drill bit

From the Fig.4, considering the stick-slip motion of the drill string under the axial vibration, when the rotary speed of the turntable is constant, since the subjected torque of the drill bit is less than the minimum required torque for rock-breaking, the drill string generates the stick-slip motion in the beginning time. And the subjected torque of the drill bit is equal to the torque generated from the torsional deformation of the drill string. When the stick-slip motion is end, the drill bit begins to generate slipping motion. The torque of the bit is equal to dynamic friction torque at the moment. Since the force that the bottom subjects from the drill bit changes periodically, the torque of the bit is also changing periodically. And the ROP begins to increase from zero to the maximum and then decreases to zero. Then the drill bit generates the stick-slip motion once, and the forces keep unchanged. Considering the stick-slip motion of the drill string substituted the axial forces, the period of torque variation is related to the rotary speed of the turntable, and it presents a decreasing tendency with the rotary speed of the turntable increasing.

#### The results contrast of existing axial vibration or not 4.

According to the foregoing, considering both torsional vibration and axial vibration, the stick-slip phenomenon about the drill string is researched. And the obtained results are more consistent with the actual working conditions of the drill string system. Based on this, using the built analysis model, The results contrast of existing axial vibration or not is conducted. Assuming that the rotary speed of turntable is written as  $\omega = 2.5\pi (rad/s)$  and other parameters is kept same, the number value is substituted into the calculation formula and then the calculation result is obtained, as shown in Fig.5, based on considering the axial vibration or not.



Fig. 5 Results comparison of bit speed considering axial vibration or not

From the Fig.5, for different rotation time of the drill bit, the time that the axial vibration is ignored is less than the time for considering the axial vibration, which indicates the axial vibration can generate hysteresis effect on the drill bit. For the stability, when the axial vibration is ignored, the speed curve of the drill bit is a smooth curve and shows a decreasing tend after the first increase, which indicates the relative stability. For the results considering the axial vibration, the speed curve of the drill bit also shows a decreasing tend after the first increase, but the amplitude is more than the results ignoring the axial vibration. The speed curve is a non-smooth curve and existing fluctuation in the changing process especially in the amplitude.

## **5.** Conclusion

Compared to existing stick-slip mechanism, the research, considering the axial vibration, improves the accuracy of the stick-slip mechanism analysis and it is better to reflect truly of stick-slip mechanism. Since considering the axial vibration, the axial force of the drill string is not a stable value, and the torque at the bit is also not a stable.

Using the established calculation method, the variation of the ROP, rotational angular displacement and torque at the bit can be obtained. With the time increasing, the ROP, rotational angular displacement and torque at the bit exist some fluctuation during the page phase. With the speed of the turntable increasing, the ROP, rotational angular displacement and torque at the bit show a decreasing gradually tend.

The results, analyzing the variation of the ROP and torque, can provide evidence for avoiding the stick-slip phenomenon and improving the life of the drill string.

## References

- Wang L, Zhao B. Dynamic modeling and control strategy for turning in place motion of a twocoaxial pendulums driven spherical inspector based on stick–slip principle[J]. Mechanism and Machine Theory, 2015, 83: 69-80.
- [2] Li Q, Dong Y, Perez D, et al. Speed dependence of atomic stick-slip friction in optimally matched experiments and molecular dynamics simulations [J]. Physical review letters, 2011, 106(12): 126101.
- [3] LI Zi-feng, ZHANG Yong-gui, HOU Xu-tian, LIU Wei-dong, etal. Analysis of longitudinal And torsion vibration of drillstring[J]. Engineering Mechanics, 2004, 21(6): 203-210.

- [4] Capozza R, Rubinstein S M, Barel I, et al. Stabilizing stick-slip friction[J]. Physical review letters, 2011, 107(2): 024301.
- [5] FENG Jian-jun, TANYuan-qian. Analysis of the Stick-Slipping Contact between a Cylinder and a Plane[J]. Natural Science Journal of Xiangtan University, 2009, 31(3): 48-53.
- [6] HUANG Gun, YIN Guang-zhi. Chaotic behavior of stick slip in rock bursts[J]. Journal of Chongqing University, 2009, 32(6): 633-637.
- [7] Amundsen D S, Scheibert J, Thøgersen K, et al. 1D model of precursors to frictional stick-slip motion allowing for robust comparison with experiments[J]. Tribology Letters, 2012, 45(2): 357-369.
- [8] Crowther A R, Singh R. Analytical investigation of stick-slip motions in coupled brake-driveline systems[J]. Nonlinear Dynamics, 2007, 50(3): 463-481.
- [9] LIU Li-lan, LIU Hong-zhao, WU Zi-ying, YUAN Da-ning, etal. Effects of the Varying Normal Force on the Stick-Slip Behavior ofM ass-Belt Systems[J]. Journal of Xi.an University of Technology, 2010 (1): 20-25.
- [10] Passelègue F X, Schubnel A, Nielsen S, et al. From sub-Rayleigh to supershear ruptures during stick-slip experiments on crustal rocks[J]. Science, 2013, 340(6137): 1208-1211.
- [11] Kovalyshen Y. Understanding root cause of stick–slip vibrations in deep drilling with drag bits[J]. International Journal of Non-Linear Mechanics, 2014, 67: 331-341.
- [12] HOU Yin-feng, WANG Zhong-min, MA Jian-min. The Stick-Slip Vibration Analysis of Non-Rotary Body Induced by Rotary Friction[J]. Journal of Xi.an University of Technology, 2006, 22(1): 78-81.
- [13] SunYongxing,LinYuanhua,ShiTaihe,etal.The Advance for Measuring the torque of Drill String[J].Petroleum Drilling Techniques, , 2005, 33(2): 13-15.
- [14] Huang Genlu, Han Zhiyong. Mechanism Analysis on Torsional Stick\_slip Vibration of Drillstring in Extended Reach Well and Some Ways to Its Suppression[J]. Petroleum Drilling Techniques, 2001, 29(2): 4-6.
- [15] Márton L, Lantos B. Modeling, identification, and compensation of stick-slip friction[J]. Industrial Electronics, IEEE Transactions on, 2007, 54(1): 511-521.
- [16] Zamanian M, Khadem S E, Ghazavi M R. Stick-slip oscillations of drag bits by considering damping of drilling mud and active damping system[J]. Journal of Petroleum Science and Engineering, 2007, 59(3): 289-299.
- [17] Zhang Guagnwei.Finite Element Analysis of Vibration Collision Characteristics of Drill Stem with the Sidewell[J]. XAPI, 1996, 11(6): 20-22.