Dynamic calculation of high-rise structures

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Abstract

When calculating the seismic response of high-rise structures, a series of dynamic response equations should be established. Because each order of array will affect each floor of the structure, a generalized coordinate system should be established to decouple the equations, and the motion equation of multi-degree-of-freedom system should be transformed into a single-degree-of-freedom system for dynamic calculation to do solution of reaction parameters. Taking the mass model of 20-storey structure as an example, the results of various reaction parameters are calculated, and the time history diagrams of displacement and acceleration are made and analyzed.

Keywords

Mode shape,generalized coordinates,decoupling assumption,Rayleigh damping.

1. Introduction

Based on the basic theoretical analysis of structural dynamics, the dynamic response parameters of multi-degree-of-freedom structural system equation are calculated. Structural isolation analysis is carried out and structural equilibrium equations are established. However, with the interaction between floors, each vibration will overlap with each other, resulting in inaccurate calculation. The generalized coordinate system is introduced to calculate the response parameters in a generalized way, and the dynamic equation is compiled. The Elsentro seismic wave [1] (EL Centro wave) is introduced to solve the inter-story displacement, floor displacement, floor acceleration and floor velocity of the structure, and the effect of each floor is compared.

2. Establishment of equilibrium equation of undamped free vibration

The mode of vibration is an inherent attribute of the structure. By calculating the frequency, the modes of vibration at different frequencies are obtained, and the 20-story structure is simplified into "gourd string" balls [2], as shown in Fig. 1.

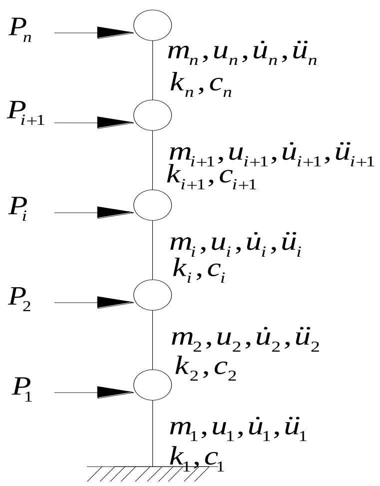

Fig.1 Structural analysis model

We can express it in matrix form:

$$[M]\{\ddot{u}\}+[K]\{u\}=\{0\} \tag{1}$$

Can be expressed as:

$$\begin{pmatrix} m_1 & 0 & \cdots & 0 \\ 0 & m_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & m_{20} \end{pmatrix}\begin{pmatrix} \ddot{u}_1 \\ \ddot{u}_2 \\ \vdots \\ \ddot{u}_{20} \end{pmatrix}+\begin{pmatrix} k_1+k_2 & -k_2 & \cdots & 0 & 0 \\ -k_2 & k_2+k_3 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & -k_{19} & 0 \\ 0 & 0 & -k_{19} & k_{19}+k_{20} & -k_{20} \\ 0 & 0 & 0 & -k_{20} & k_{20} \end{pmatrix}\begin{pmatrix} u_1 \\ u_2 \\ \vdots \\ u_{19} \\ u_{20} \end{pmatrix}=\begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \tag{2}$$

Let $\{u(t)\}=\{\phi\}\sin(wt+\theta)$, substitute into equation (1)

Solve this equation:

$$([K]-w^2[M])\{\phi\}=\{0\} \tag{3}$$

Then its matrix formula is:

$$\left\|[K]-w^2[M]\right\|=\begin{vmatrix} (k_1+k_2)-w^2m_1 & -k_2 & \cdots & 0 & 0 \\ -k_2 & (k_2+k_3)-w^2m_2 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & -k_{19} & 0 \\ 0 & 0 & -k_{19} & (k_{19}+k_{20})-w^2m_{19} & -k_{20} \\ 0 & 0 & 0 & -k_{20} & k_{20}-w^2m_{20} \end{vmatrix}=0$$

Since the mass and stiffness of the 20-storey building are invariants determined by their own properties, the mass matrix and stiffness matrix can be obtained, and the known quantities k and m are substituted into the above matrix to obtain the equation of the 20th order with w^2, from which 20 positive roots of w^2 can be obtained, w can be calculated, and 20 natural vibration frequencies can be obtained by eliminating the negative roots. By substituting into equation (3), modes of vibration at different natural frequencies can be obtained.

The inherent mode of the system is:

$$[\phi]=\begin{pmatrix} \phi_1^{(1)} & \phi_1^{(2)} & \cdots & \phi_1^{(20)} \\ \phi_2^{(1)} & \phi_2^{(2)} & \cdots & \phi_2^{(20)} \\ \vdots & \vdots & \ddots & \vdots \\ \phi_{20}^{(1)} & \phi_{20}^{(2)} & \cdots & \phi_{20}^{(20)} \end{pmatrix} \tag{4}$$

3. Solution of motion equation

The forced motion equation of the damped multi-free system is as follows [3] :

$$[M]\{\ddot{u}\}+[C]\{\dot{u}\}+[K]\{u\}=\{P\} \tag{5}$$

Make the following coordinate transformation, let $\{u\}=[\phi]\{q\}$, and then multiply left $[\phi]^T$, get:

$$[\phi]^T[M][\phi]\{\ddot{q}\}+[\phi]^T[C][\phi]\{\dot{q}\}+[\phi]^T[K][\phi]\{q\}=[\phi]^T\{P\} \tag{6}$$

Then through the orthogonality of the mode, we can get:

$$\{\phi\}_i^T[M]\{\phi\}_j=\{0\} \tag{7}$$

Get:

$$[\phi]^T[M][\phi]=\begin{pmatrix} M_1 & 0 & \cdots & 0 \\ 0 & M_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & M_{20} \end{pmatrix} \tag{8}$$

Among them $M_i=\{\phi\}_i^T[M]\{\phi\}_i$ It can be known that equation (8) is a diagonal matrix. Similarly, $[\phi]^T[K][\phi]$ is also a diagonal matrix. However, $[\phi]^T[C][\phi]$ cannot be guaranteed to be a diagonal matrix. When there is damping in the system, for the convenience of analysis, $[\phi]^T[C][\phi]$ is approximately regarded as a diagonal matrix, which is called the decoupling assumption.

Assuming that $M_i=\{\phi\}_i^T[M]\{\phi\}_i$; $C_i=\{\phi\}_i^T[C]\{\phi\}_i$; $K_i=\{\phi\}_i^T[K]\{\phi\}_i$; $P_i=\{\phi\}_i^T[P]$

The dynamic response matrix equation of the regular transformation is:

$$[\phi]^T[M][\phi]\{\ddot{q}\}+[\phi]^T[C][\phi]\{\dot{q}\}+[\phi]^T[K][\phi]\{q\}=[\phi]^T\{P\} \tag{9}$$

The decoupling system of transformation is:

$$M_i\ddot{q}_i+C_i\dot{q}_i+K_iq_i=P_i \quad (i=1,2,\cdots,20) \tag{10}$$

In this way, the problem of solving complex matrix equations is transformed into a problem of solving linear differential equations with second-order constant coefficients. The problem of solving the equations of the motion system of a multi-free system is transformed into the problem of solving the motion system of a single-degree-of-freedom system.

Divide both sides of the equation by M_i, the damping coefficient determined by the single-degree-of-freedom system is $C_i=2\zeta_i w_i M_i$, and substitute $w_i^2=\dfrac{K_i}{M_i}$ into it, then the equation can be simplified as:

$$\ddot{q}_i+2\zeta_i w_i\dot{q}_i+w_i^2q_i=\frac{1}{M_i}P_i \quad (i=1,2,\cdots,20) \tag{11}$$

It can be seen that the generalized displacement, generalized velocity and generalized acceleration of the structure can be determined by selecting appropriate damping. Rayleigh damping is selected in the form of $[C]=a_0[M]+a_1[K]$, where a_0 and a_1 are the proportionality coefficients. Then, the Duhamel integral method or runge-kutta integral method is used to solve $q_i(t)$[4]. Considering the non-zero initial solution, the specific solution $q_i^0(t)$ is determined, and then the displacement at time t is calculated as follows:

$$u(t)=\sum_{i=1}^{20}\{\phi\}_i\left[q_i^0(t)+q_i(t)\right] \tag{12}$$

4. Determination of structural parameters

The building model is simplified into a discrete multi - degree of freedom system with 20 concentrated masses. The structure parameter is about $m_i=2933t$, $k_i=28950000\ kN/m$. According to the specification, the structural mode damping ratio of $\zeta=0.05$ can be determined. Rayleigh damping is adopted, the first two natural frequencies of the structure are w_1 and w_2, and $a_0=0.5705$ and $a_1=0.0033$ can be calculated, so that the damping is about $c_i=22000kN\cdot s\cdot m^{-1}$. For site category II class.

5. Analysis and comparison of results

Through the calculation of the parameter model, the natural frequency of the structure is obtained as shown in table 1:

Table1 Natural frequency of structural model

Order number	1	2	3	4	5	6	7	8	9	10
Frequency/Hz	1.21	3.63	6.02	8.38	10.69	12.94	15.11	17.19	19.17	21.04

The structure feeds the 40s EL Centro seismic waves, and adjusts the peak acceleration of the waves to $u_g = 4$ m/s2. The earthquake intensity is equivalent to 8 degrees [5]. Its waveform is shown in fig 2:

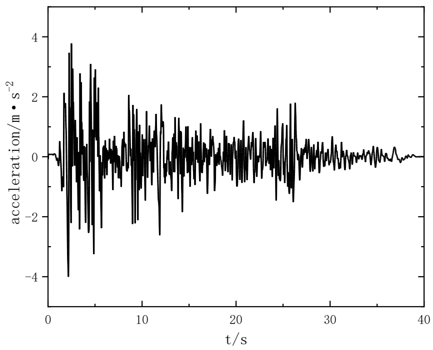

Fig.2 EL Centro

The program calculated the maximum dynamic parameters as shown in table 2:

Table2 Results of dynamic response parameters of each floor

Parameter	Displacement between the	Floor displacement（m	Floor rate（m/s）	Floor acceleration
0-1(1)	23.62	0.02	0.19	2.61
1-2(2)	23.34	0.05	0.38	5.12
2-3(3)	22.86	0.07	0.55	7.39
3-4(4)	22.28	0.09	0.71	9.42
4-5(5)	21.63	0.11	0.85	11.36
5-6(6)	20.84	0.13	0.97	12.90
6-7(7)	19.83	0.15	1.07	13.99
7-8(8)	18.59	0.17	1.16	14.69
8-9(9)	17.13	0.19	1.25	16.06
9-10(10)	15.57	0.21	1.37	17.25
10-11(11)	14.06	0.22	1.52	17.83
11-12(12)	13.01	0.23	1.67	17.75
12-13(13)	12.18	0.24	1.81	17.12
13-14(14)	11.30	0.25	1.94	16.17
14-15(15)	10.26	0.26	2.04	15.36
15-16(16)	9.00	0.26	2.13	15.58
16-17(17)	7.53	0.27	2.20	16.26
17-18(18)	5.84	0.27	2.25	17.90
18-19(19)	3.99	0.28	2.29	19.20
19-20(20)	2.02	0.28	2.31	19.96

Use Origin to draw the results of all parameters in table 2, as shown in figure 3:

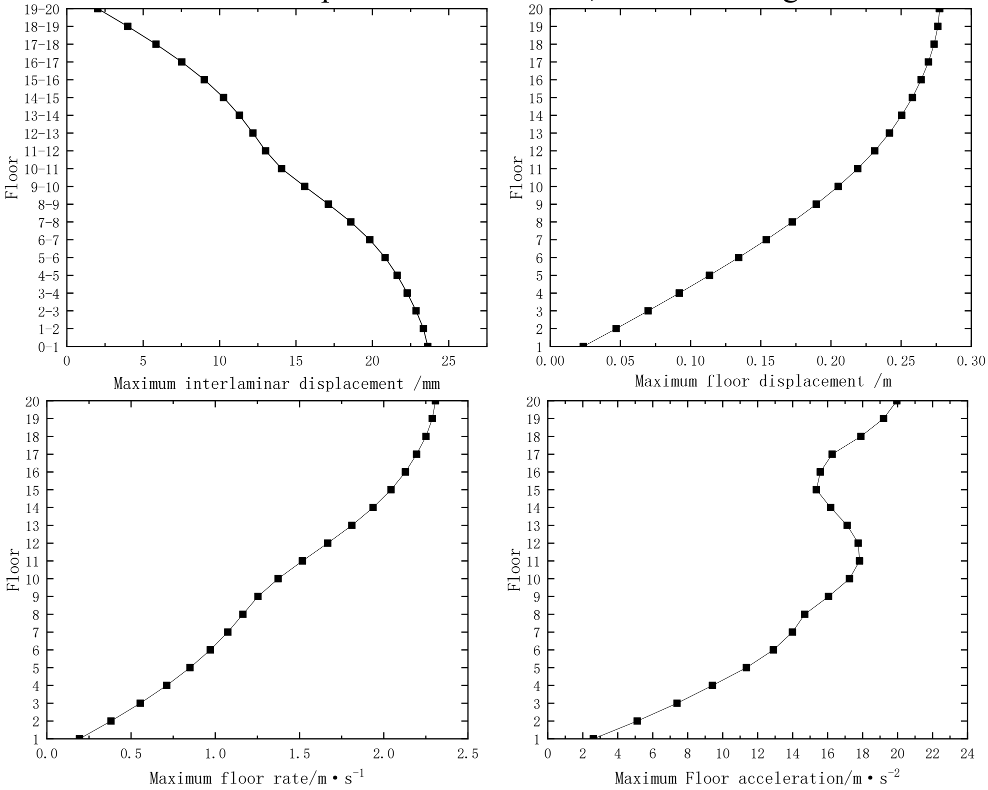

Fig.3 Dynamic reaction result analysis diagram

By analyzing table 2 and figure 3, it can be concluded that, with the higher the number of floors, the smaller the inter-layer displacement is, the larger the floor displacement is, and the larger the velocity is. The acceleration will be maximum at the middle level and the top level. This is consistent with real life.

Below, the structural displacement and acceleration of the model structure are analyzed to analyze the time history, as shown in Fig. 4:

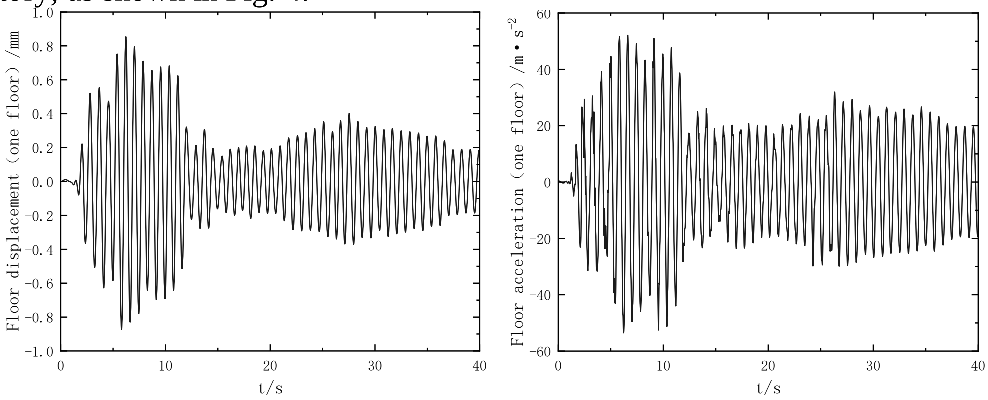

Fig.4 Dynamic reaction time analysis diagram

Compared with figure 4 and figure 2, when the acceleration of seismic wave in figure 2 increases, the floor displacement and floor acceleration also increase relatively. Observation data, the earthquake in the 40s of time, the maximum horizontal displacement of the structure will be in a short span of 6 s time increases to 0.8 m, the instantaneous acceleration of speed of going to 50 m/s2, the collapse of buildings, the damage will be inevitable, so shock absorption study, join the damper actuator, the dynamic responses will value decreases, and is the indispensable means for seismic.

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