X-ray pulsar signal de-noising method based on wavelet domain

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Abstract

The denoising process of X-ray pulsar signal largely determines the accuracy of pulsar navigation. The traditional wavelet domain denoising method can not accurately screen out reasonable wavelet base and does not consider the impact of the noise wavelet coefficient with decomposition layer. Many shortcomings of the threshold function make it not only unable to meet the accuracy requirements of pulsar navigation, but also reduce the realtime of navigation. Based on this, this paper proposes a method to screen the optimal wavelet base and the optimal decomposition layer based on the cross-correlation coefficient, and constructs a threshold function based on decomposition layer. The experimental results show that compared with the traditional wavelet domain denoising method, this method can not only accurately and quickly select the optimal wavelet base and optimal decomposition layer suitable for pulsar signals, but also using the threshold function constructed in this paper to denoise the noisy puplsar can significantly improve the signal-to-noise ratio, peak signal-to-noise ratio and peak position error, provide a new idea for pulsar signal denoising.

Keywords

Pulsar, signal denoising, wavelet transform, cross correlation coefficient, threshold function

1. Introduction

X-ray pulsar navigation technology is a new autonomous navigation technology, which measures the time of arrival of pulsar X-ray photons to provide time, position, attitude and speed measurement for spacecraft[1]. However, because pulsars are very far away from the solar system, their photon radiation intensity is extremely weak, and they are also affected by many noises from cosmic rays, making it difficult to obtain X-ray pulsar signals with high signal-to-noise ratio.

The pulsar signal is a typical non-stationary signal. Wavelet transform has the advantages of multi-resolution time-frequency analysis, and is particularly suitable for processing non-stationary signals[2]. Usually in the wavelet domain, a wavelet basis function is selected, the noisy signal is decomposed at different scales to obtain wavelet decomposition coefficients, and then soft and hard threshold functions are used to process the coefficients and then reconstruct the coefficients to achieve the purpose of noise reduction. In terms of wavelet base selection: Lu Guangsen et al.[3] proposed to use an exhaustive method to denoise the noisy signal in turn, and use the wavelet base and decomposition layer under the maximum signal-to-noise ratio as the denoising wavelet base and decomposition layer, and then reuse it This wavelet base and decomposition layer denoise the signal. The disadvantage of this method is that the cart before the horse is denoised and then the wavelet base is selected. In addition, the wavelet base is determined only by the index of the optimal signal-to-noise ratio. The method is not reasonable. In terms of threshold function construction: Wang Yongkai et al.[4] constructed a dual-parameter threshold function to denoise noisy signals based on the defects of traditional soft and hard threshold functions. The disadvantage of this method is that the denoising effect is largely Depending on the selection of parameters, the paper proposes to use an exhaustive method to select parameters. Although the denoising performance is softer and the hard threshold function is greatly improved, it cannot meet the real-time requirements of pulsar navigation due to the multiplication of calculations. In addition, for pulsar signal denoising, weak changes in parameters will lead to a sharp decrease in navigation accuracy, and even its signal-to-noise ratio is not as good as traditional soft and hard threshold denoising methods[5].

Based on the above analysis, this article firstly determines the optimal wavelet base and optimal decomposition layer according to the cross-correlation degree between the wavelet decomposition coefficients under the wavelet bases of the noisy signal and the standard pulsar signal contour coefficient. Considering that the noise wavelet decomposition coefficients are decomposed The characteristic of the increase and decrease of the layer proposes to construct a threshold function model based on the decomposition layer to denoise the noisy signal. By comparing the method in this paper with the soft threshold function, hard threshold function, and parameter-containing threshold function, it can be concluded that the method in this paper can not only reduce the extra calculation amount brought by the calculation parameters, but also effectively retain the peak information in the pulsar signal profile. Such detailed features are greatly improved in the three indicators of signal-to-noise ratio (SNR), peak position error (EPP), and peak signal-to-noise ratio (PSNR).

2. Selection of wavelet basis and decomposition layer based on cross-correlation coefficient

2.1 Basic Ideas of Denoising X-ray Pulsar Signals in Wavelet Domain

A noisy pulsar signal model can be expressed as follows:

$$y(t) = x(t) + s(t) \tag{1}$$

Among them, $y(t)$ is the noisy pulsar signal, $x(t)$ is the standard pulsar signal, and $s(t)$ is the noise signal. Since wavelet transform is a linear transform, after discrete wavelet transform is performed on the signal, the relationship between the wavelet decomposition coefficients is:

$$w_{j,k} = u_{j,k} + v_{j,k} \tag{2}$$

Since the energy of the standard signal is concentrated in the wavelet domain, and the energy of the noise is distributed in the entire wavelet domain, the wavelet decomposition coefficient $u_{j,k}$ corresponding to $x(t)$ is usually larger than the wavelet decomposition coefficient $v_{j,k}$ corresponding to $s(t)$. By reasonably selecting the threshold Thr, set the wavelet coefficients smaller than the threshold to 0, and keep the wavelet coefficients larger than the threshold directly (hard threshold function), or shrink to 0 by a fixed amount (soft threshold function) to achieve the purpose of denoising[6].

2.2 Selection of wavelet basis and decomposition layer based on cross-correlation coefficient

The principle of wavelet transform is to use the wavelet base to extract the time-frequency information of the signal. If the wavelet base is highly correlated with the signal, the more accurate the extracted information will be. This accuracy can be reflected by wavelet decomposition coefficients. The wavelet coefficient describes the degree of similarity between the wavelet base and the signal in each high and low frequency band. Usually a noisy X-ray pulsar signal, the real signal is always in the low frequency band, and although the noise signal is distributed in each frequency band, the low frequency noise is easily covered by the real signal, so the detected noise signal is usually located in High frequency band. Therefore, in the analysis of the cross-correlation coefficient, the wavelet coefficients in the high frequency band can be omitted, and only the low frequency band coefficients associated with the real signal, that is, the contour coefficients, can be extracted.

The prerequisite for the calculation of the cross-correlation coefficient is that the data lengths of the two signals are the same, but when using Matlab to perform wavelet transformation on the signal, the data length of the contour coefficient in the low frequency band always changes according to the number of decomposition layers, and its length is always smaller than the standard signal Data length, therefore, it is necessary to up-sample the low frequency band data until it is the same as the standard signal data length. This article selects the resample function in Matlab for up-sampling processing, and its functional formula is:

$$x = resample\left(\mathrm{x}, length\left(y\right), length\left(x\right)\right) \tag{3}$$

Its meaning is that the contour data x is up-sampled, the data length of the contour coefficient is changed from length(x) to length(y) by the function resample, and then the contour coefficient x of the changed data length and the real signal y are mutually determined. Number of relations. Its mathematical expression is:

$$P = \frac{\mathrm{cov}(x,y)}{\delta_x \delta_y} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(\mathrm{y}_i - \bar{y})}{n * \delta_x \delta_y} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(\mathrm{y}_i - \bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2 * \sum_{i=1}^{n}(\mathrm{y}_i - \bar{y})^2}} \tag{4}$$

In the formula, P is the cross-correlation coefficient, x is the low-frequency profile data with the changed data length, y is the real signal data, $\mathrm{cov}(x,y)$ is the covariance between the two signal variables, δ_x, δ_y is the standard deviation of the two signal variables, and x_i, y_i is the value of the two signal variables at i and $\bar{x}, \bar{y}$ is the average value.

3. Threshold function construction based on decomposition layer parameters

3.1 Selection of Threshold Function in Wavelet Threshold Denoising

Traditional threshold functions include hard threshold functions, soft threshold functions and threshold functions with parameters. Its mathematical expression is:

Hard threshold function expression:

$$\hat{w}_{j,k} = \begin{cases} w_{j,k}, & \left|w_{j,k}\right| \ge Thr \\ 0, & \left|w_{j,k}\right| < Thr \end{cases} \tag{5}$$

Soft threshold function expression:

$$\hat{w}_{j,k} = \begin{cases} \mathrm{sgn}(w_{j,k})(\left|w_{j,k}\right| - \mathrm{Thr}), & \left|w_{j,k}\right| \ge Thr \\ 0, & \left|w_{j,k}\right| < Thr \end{cases} \tag{6}$$

Threshold function expression with parameters:

$$\hat{w}_{j,k} = \begin{cases} \mathrm{sgn}(w_{j,k})(\left|w_{j,k}\right| - \frac{(1-\mathrm{a}) * \mathrm{Thr}}{e^{(\left|w_{j,k}\right| - Thr)}} - \frac{a * Thr}{e^{b(\left|w_{j,k}\right| - Thr)}}) \\ 0, \end{cases} \tag{7}$$

Among them, the hard threshold function is to set the wavelet coefficients smaller than the threshold to 0, and the wavelet coefficients larger than the threshold are not processed. Due to the "one size fits all" processing method, the wavelet coefficients will inevitably cause sudden changes at the threshold, resulting in the denoised signal The discontinuity. The soft threshold function compensates for the discontinuity of the hard threshold processing method, but because it always attenuates the wavelet coefficients at a fixed value when the threshold is greater than the threshold, the signal always produces a constant deviation after denoising. Although the parameter-containing threshold function construction method makes the signal continuous after denoising and the denoising effect is significantly improved compared with the soft and hard threshold function, it does not give specific instructions on the choice of parameters. Considering that the wavelet coefficient of noise decreases with the increase of the number of decomposition layers, the constructed threshold function also needs to take the number of decomposition layers into consideration.

3.2 Threshold function based on decomposition layer parameters

Through the analysis of the characteristics of soft, hard, and parameter-containing threshold functions, the threshold function constructed in this article should be considered from the following three aspects: 1. The threshold function is continuous at the threshold and there is no constant deviation above the threshold; 2. To reduce calculations The amount of denoising meets the real-time requirements and can have good denoising stability for different pulsar signals. The threshold function does not set uncertainty parameters; third, the construction of the threshold function should be related to the number of wavelet decomposition layers, and the construction The result is that with the increase of the decomposition layer, it is closer to the wavelet coefficients of the real signal. Based on this, the constructed threshold function expression is

$$\hat{w}_{j,k} = \begin{cases} \operatorname{sgn}(w_{j,k})\sqrt[j]{|w_{j,k}|^j - Thr(j)^j} & |w_{j,k}| \geq Thr(i) \\ 0 & |w_{j,k}| < Thr(i) \end{cases} \tag{8}$$

In the formula, $w_{j,k}$ represents the wavelet coefficient at the position k of the j-th decomposition layer, $\hat{w}_{j,k}$ is the estimated value, $\operatorname{sgn}(w_{j,k})$ is the sign of the wavelet coefficient at that place, $|w_{j,k}|$ is the absolute value of the wavelet coefficient at that place, and $Thr(j)$ is the threshold of the j-th decomposition layer.

From the constructed threshold function expression, we know that when $|w_{j,k}| \to Thr(j)$, $\hat{w}_{j,k} \to 0$, the expression approaches the soft threshold function, when $\hat{w}_{j,k} \to \infty$, $\hat{w}_{j,k} \to w_{j,k}$, the expression approaches the hard threshold function.

The relationship between this threshold function and the number of decomposition levels is discussed below:

Suppose $w_{j,k} \geq Thr(j)$ and $w_{j,k}$ are continuous, $1 \leq j \leq 10$ and continuous, since the pulsar signal threshold is always greater than 0, that is, $Thr(j) > 0$, then $w_{j,k} > 0$ is continuous, and $y = \sqrt[j]{w_{j,k}{}^j - Thr(j)^j}$ is derived from decomposition layer j:

Take the logarithm of both sides of the equation:

$$\ln y = \frac{1}{j}\ln(w^j - Thr^j) \tag{9}$$

Also seek derivation:

$$\frac{y'}{y} = \frac{1}{j}\frac{w^j \ln(w^j) - Thr^j \ln(Thr^j)}{w^j - Thr^j} - \frac{1}{j^2}\ln(w^j - Thr^j) \tag{10}$$

Simplified:

$$y' = (\sqrt[j]{w^j - Thr^j}) * (\frac{1}{j}\frac{w^j \ln(w^j) - Thr^j \ln(Thr^j)}{w^j - Thr^j} - \frac{1}{j^2}\ln(w^j - Thr^j)) \tag{11}$$

Due to $w_{j,k} \geq Thr(j)$, and

$$\frac{1}{j}\frac{w^j \ln(w^j) - Thr^j \ln(Thr^j)}{w^j - Thr^j} > \frac{1}{j^2}\ln(w^j - Thr^j), \text{ so } y' > 0 \tag{12}$$

If the derivative is greater than 0, the original function increases monotonously. It can be seen that the estimated wavelet coefficient $\hat{w}_{j,k}$ is closer to the noisy wavelet coefficient $w_{j,k}$ as the number of decomposition layers increases, because the proportion of the wavelet coefficient component of noise in $w_{j,k}$ decreases with the increase of the decomposition layer. Is small, so that the proportion of wavelet coefficients generated by real signals in the high resolution layer is much higher than that of noise wavelet coefficients, this makes $\hat{w}_{j,k}$ closer to the real signal value. At the same time, it can be seen from the above derivation that the constructed threshold function is continuous and derivable, which is of great significance for subsequent mathematical applications.

4. Experimental results and analysis

This paper selects the X-ray photon pulse signal radiated by the pulsar PSR B0531+21 for experiments. The observation data comes from the pulsar observation database of the Parks Observatory in Australia. The observed X-ray photon pulse sequence is epoch-folded to obtain the contour of the noisy pulsar signal ,As shown in Figure 1. The standard profile of the pulsar signal comes from the European Pulsar Database (EPN), as shown in Figure 2.

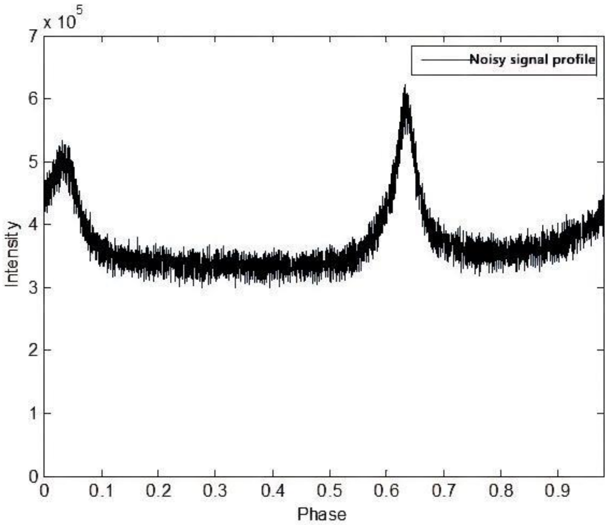

Fig 1: Noisy signal profile of PSR B0531+21

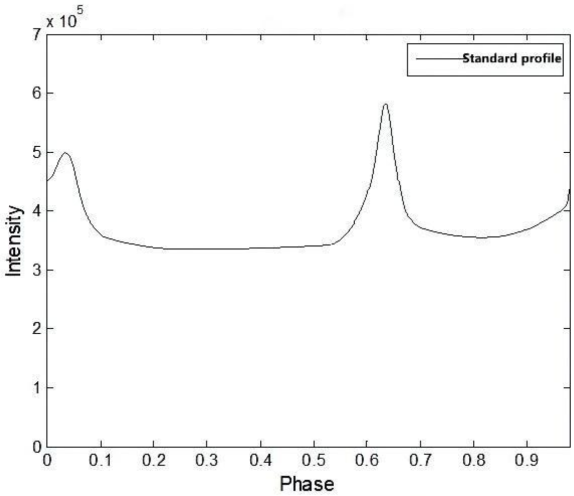

Fig 2: Standard signal profile of PSR B0531+21

First, determine the optimal wavelet base and the number of decomposition levels according to the correlation coefficient method. From the correlation coefficients in Figures 3 to 5 and the three-dimensional scatter plot of the wavelet base and decomposition levels, we can see that sym7 with a correlation coefficient of 0.923 is selected as the optimal wavelet In this case, the number of decomposition layers is 5, and the threshold function proposed in this paper is used to denoise the noisy pulsar signal under the threshold criterion based on the estimated mean square error of the noise of each layer. At the same time, soft and hard threshold functions, A control experiment was carried out with the parameter threshold function, and the denoising pulse contour diagrams in Fig. 6 to Fig. 10 were obtained.

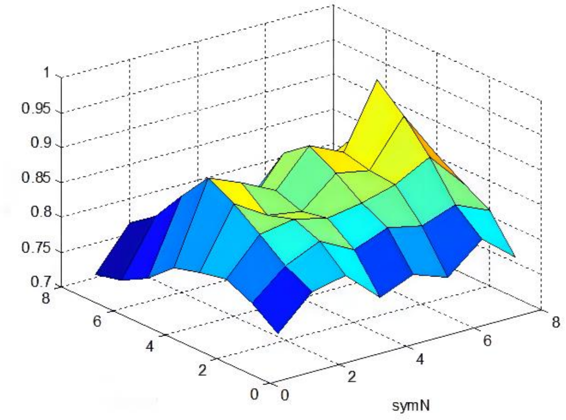

Fig 3: Correlation coefficient distribution of sym series wavelet under 1~8 decomposition layer

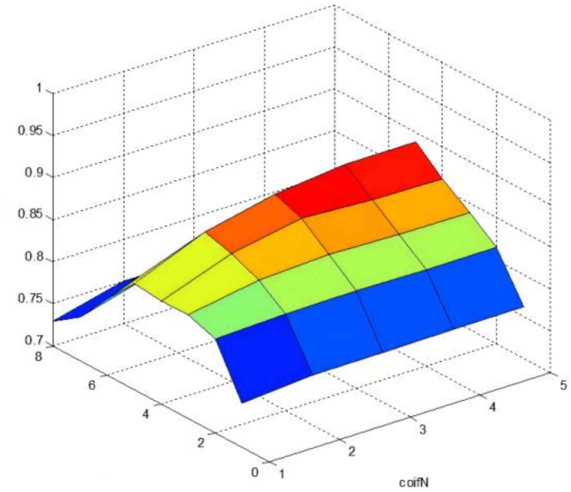

Fig 4: Correlation coefficient distribution of coif series wavelet under 1~8 decomposition layer

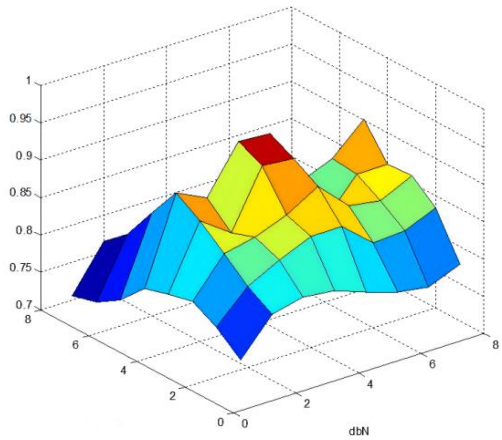

Fig 5: Correlation coefficient distribution of db series wavelet under 1~8 decomposition layer

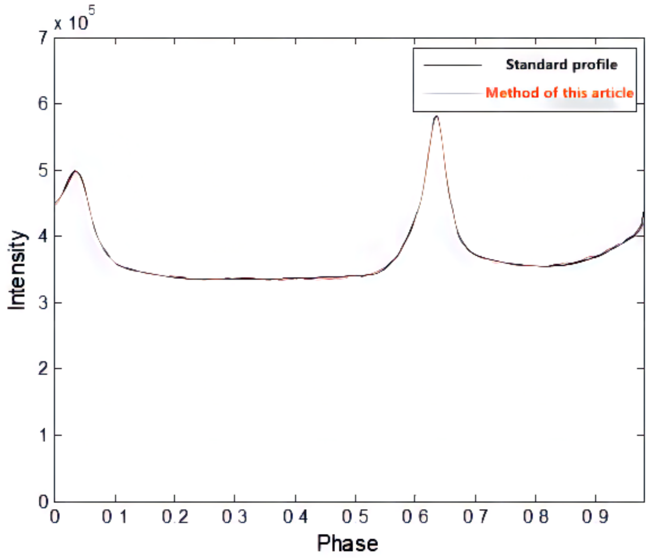

Fig 6: Comparison of the denoising profile of this paper and standard profile

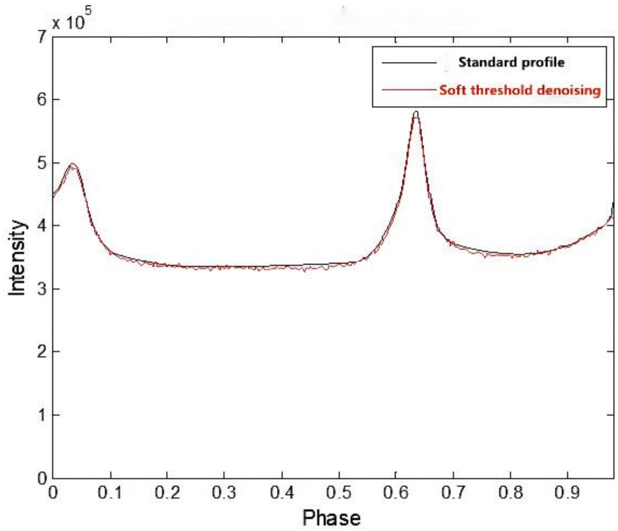

Fig 7: Comparison of the denoising profile of soft threshold method and standard profile

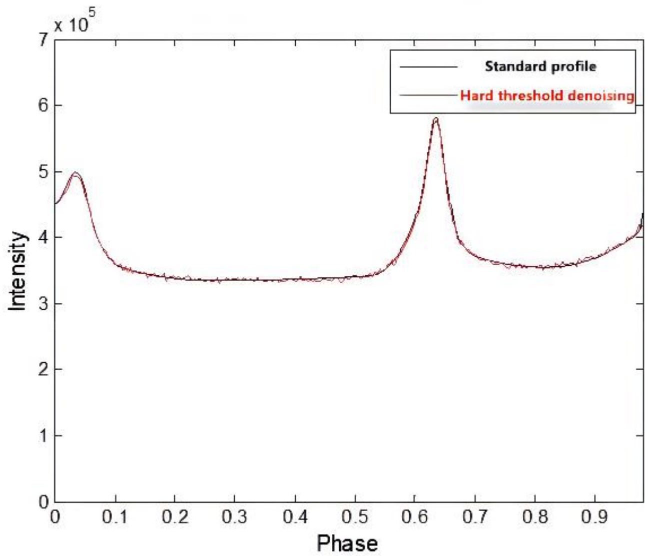

Fig 8: Comparison of the denoising profile of hard threshold method and standard profile

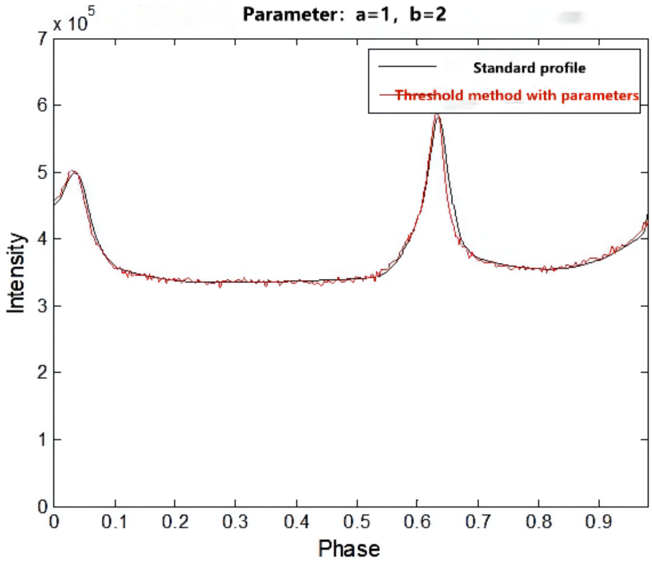

Fig 9: Comparison of the denoising profile of parameter threshold method and standard profile at the a=1,b=1

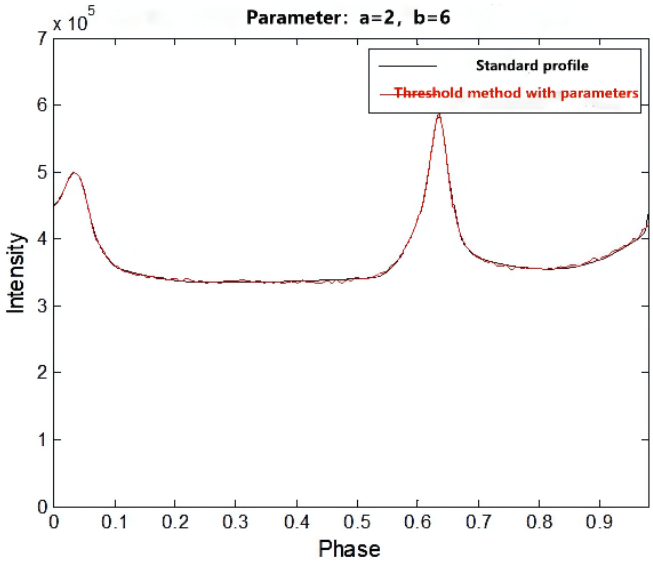

Fig 10: Comparison of the denoising profile of parameter threshold method and standard profile at the a=1,b=1

It can be seen from Figure 7 that the signal after denoising with soft threshold has a constant threshold difference between its wavelet coefficients and the original signal, so the contour of the signal after denoising is relatively lower after hard threshold processing. , Which makes the peak error larger. Because the hard threshold function adopts a "one size fits all" strategy for wavelet coefficients smaller than the threshold, the signal contour after hard threshold processing is not smooth, as shown in Figure 8, and the wavelet coefficients larger than the threshold are not processed, which is not consistent with the actual situation. This leads to a small improvement in the signal-to-noise ratio. For the threshold function containing parameters, if the parameters are not selected properly, even if the optimal wavelet basis and the optimal decomposition layer are used, the denoising contour will be unsatisfactory, even inferior to the denoising of soft and hard threshold functions, as shown in Figure 9. When the parameters a=1.3 and b=4.3, after denoising by the parameter-containing threshold function, the contour and the standard contour are compared. The peak error and signal-to-noise ratio are worse than the soft and hard threshold functions, and the peak position error is larger. Cannot be applied to subsequent navigation. Figure 10 shows the signal profile after denoising when a=2.4 and b=7.8. Although it is not as good as this method in the smooth section, its peak position error and peak signal-to-noise ratio are equivalent to this method. It can be seen that the selection of the parameters of the parameter-containing threshold function is critical and determines the denoising performance to a large extent. However, considering the real-time and accuracy requirements of pulsar navigation, the exhaustive method, simulated annealing method, The ant colony algorithm and the bee colony algorithm are not suitable. The comparison between the denoising contour of the method in this paper and the standard contour is shown in Figure 6. Although the edge effect is the same as other denoising methods, the denoising effect of the method in this paper is very good compared with other methods in both the peak and smooth sections. Great improvement.

In order to further verify the denoising performance of the method in this paper, the signal-to-noise ratio (SNR), peak signal-to-noise ratio (SNR), and peak position error (EPP) are selected as evaluation indicators. The definition of each indicator is as follows:

$$SNR = 10 * \lg\left(\frac{\sum x^2}{\sum (x - \hat{x})^2}\right) \tag{13}$$

$$PSNR = 10 * \lg\left(n * \frac{\max(x)^2}{\sum (x - \hat{x})^2}\right) \tag{14}$$

$$EPP = \sqrt{(P_O - P_d)^2} \tag{15}$$

In the formula, x is the standard pulsar signal data, $\hat{x}$ is the pulsar signal data after denoising, n is the length of the data, P_o is the position of the standard pulse contour pulse peak, and P_d is the position of the pulse contour pulse peak after denoising. It is easy to know that the larger the SNR and PSNR and the smaller the EPP, the better the denoising effect.

In the experiment, db4 with a maximum correlation coefficient of 0.906 and coif5 with a correlation coefficient of 0.875 were selected from db and coif as control experiments. At this time, their best decomposition levels are all 5 layers. For these three wavelet bases, use the method, soft threshold, hard threshold, and parameter-containing threshold denoising method to denoise the noisy pulsar signal under their corresponding decomposition layers. Three evaluation indicators under different denoising methods As shown in Table 1.

Table 1: Comparison of various indicators after denoising by different methods

Evaluation index	Method of this article	Soft threshold	Hard threshold	Threshold with parameters (a=1,b=3)	(a=4,b=5)
SNR/dB	38.31	33.20	33.39	28.97	36.29
PSNR/dB	49.92	35.81	39.42	34.37	46.69
EPP/S*10^-6	0.792	3.941	2.364	5.478	0.641

As can be seen in Table 1, the sym7 wavelet base selected by the correlation coefficient is decomposed in 5 layers under the threshold function constructed in this paper, and the best denoising effect can be obtained and the best performance index can be obtained. Because the threshold function constructed in this article is related to the decomposition layer, the wavelet coefficients of the noise are rapidly reduced after threshold denoising, while the proportion of the wavelet coefficients of the signal increases. This performance is particularly obvious at the peak of the signal, so after the method of this article is processed Compared with other methods, the peak signal-to-noise ratio and peak position error of the signal are significantly improved. Among them, the SNR is generally increased by 4 to 8dB, the softer and harder threshold functions of PSNR are improved by 39.11% and 26.52%, respectively, while the EPP is softer and harder. The threshold function is increased by 79.95% and 66.53%. Although the EPP index of the parameter-containing threshold function will be increased by about 20% compared with the method in this paper when the parameter-containing threshold value is properly selected, the calculation amount is much higher than that of the method in this paper. Therefore, the method in this paper is more It is suitable for denoising of pulsar signals.

5. Conclusion

Aiming at using wavelet transform to solve the problem of determining the wavelet base and decomposition layer in pulsar signal denoising and the construction of threshold function, this paper first analyzes the cross-correlation relationship between wavelet decomposition coefficients and signal contour coefficients to select the optimal wavelet basis and decomposition The feasibility of the layer, and construct the expression of the correlation coefficient between them, and then consider the characteristic that the noise wavelet coefficient decreases with the increase of the decomposition layer, and aim at the traditional soft threshold function, hard threshold function, and parameter-containing threshold function. Insufficient, a threshold function based on the parameters of the decomposition layer is constructed. The experimental results show that the optimal wavelet basis and the optimal decomposition layer selected by the cross-correlation method have wide applicability. Compared with the threshold function containing parameters, the constructed threshold function can not only improve the denoising stability, but also reduce the calculation To meet the real-time requirements of pulsar navigation. Compared with other de-noising methods, the method in this paper is significantly improved in terms of signal-to-noise ratio, peak signal-to-noise ratio, and peak position error, which provides a basis for the research of X-ray pulsar signal processing.

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