Comparison of Two Variable Density Methods based on Proportional Topology Optimization

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Abstract

This paper exploits two variable density methods based on proportional topology optimization (PTO) to solve the topology structure optimization problems. These two methods can solve different problems with minimum volume or minimum compliance. Also, the present methods in this paper has a high application value. In order to contrast the difference between the SIMP method and RAMP method, one numerical example is investigated to provide to observe their differences.

Keywords

Topology Optimization; SIMP Method; RAMP Method; Von Mises Stress.

1. Introduction

Topology optimization can achieve the best bearing effect with the least material, so it has been widely applied to practical engineering. Many experts developed some advanced methods which are applied to practical engineering problems [1-4]. Also, many experts have proposed a great deal of topology optimization methods. Bendsøe [5] proposed the homogenization method that was universally applied to the optimization of microstructure. The SIMP method [6] and RAMP method [7-8] are variable density methods with continuous variables. The evolutionary structural optimization (ESO) method [9], Bi-directional ESO (BESO) method [10], and sequential element rejections and admissions (SERA) method [11] are variable density methods with discrete variables. The independent continuous mapping (ICM) method [12] isa variable-density method implemented by the continuous conversion of continuous and discrete variables. The level set method (LSM) [13] was proposed based on partial differential equation. The moving morphable components (MMC) method [14] and moving morphable void (MMV) method [15] were proposed based on the topological description functions of the components. The proportional topology optimization (PTO) method [16] is a non-sensitivity method for topology. Whether it is the minimum volume problem under stress constraints or the minimum compliance problem under volume constraints, the PTO method can always solve it.

This paper adopted the SIMP method and RAMP method to calculate the Young's modulus, and the topology optimization model is solved by the proportion topology optimization method. The results obtained by the SIMP method and RAMP method are contrast with each other, and their performances are compared.

2. Basic theory

There are two optimization models of PTO method. One is to take the minimum volume as the objective function and the Von-Mises stress as the constraint condition. The other is to take the minimum compliance as the objective function and the volume as the constraint condition.

(1) Stress constrained problem. The objective function of this kind of problem is the minimum mass of the structure and the constraint is that the stress is less than the allowable stress measure.

$$\min \sum_{i=1}^{n} \rho_i v_i$$
$$\text{subject to} \begin{cases} \boldsymbol{KU} = \boldsymbol{F} \\ \sigma_i \le \sigma_{\lim}, if \ \rho > 0 \\ 0 \le \rho_{\min} \le \rho_i \le \rho_{\max} \le 1 \end{cases} \tag{1}$$

where ρ_i denotes the density of element, ρ_{min} and ρ_{max} denotes the lower bound and upper bound of density respectively; v_i denotes the volume of element; $\boldsymbol{K}$ denotes the stiffness matrix; $\boldsymbol{U}$ denotes the displacement vector; $\boldsymbol{F}$ denotes the load vector; σ_i is the element stress; σ_{lim} is the stress measure.

Based on the theory of PTOs method, the optimized element density ρ_i^{opt} can be obtained as

$$\rho_i^{\text{opt}} = \frac{M_{\text{remain}}}{\sum_{j=1}^{N} \sigma_j^q v_j} \sigma_i^q \tag{2}$$

where M_{remain} denotes the remaining material amount; q denotes the proportion exponent.

(2) Minimum compliance problem. The objective function of this problem is the minimum compliance and the constraint is that the total mass is equal to preset mass.

$$\min C = \boldsymbol{U}^{\text{T}} \boldsymbol{KU}$$
$$\text{subject to} \begin{cases} \boldsymbol{KU} = \boldsymbol{F} \\ \sum_{i=1}^{n} \rho_i v_i = M \\ 0 \le \rho_{\min} \le \rho_i \le \rho_{\max} \le 1 \end{cases} \tag{4}$$

Where C denotes the compliance; M denotes the total mass.

Based on the theory of PTOc method, the optimized element density ρ_i^{opt} can be obtained as

$$\rho_i^{\text{opt}} = \frac{M_{\text{remain}}}{\sum_{j=1}^{N} C_j^q v_j} C_i^q \tag{5}$$

In order to assure the stability of the iteration process, parameter α is adopted. Furthermore, the update formula of element density is given as

$$\rho_i^{\text{new}} = \alpha \rho_i^{\text{pre}} + (1 - \alpha) \rho_i^{\text{opt}} \tag{3}$$

In general, the value of α is set to 0.5. After actual debugging, we find that: if the value of α is less than 0.5, we could not obtain the ideal structure.

Whether it is the volume minimum model or the compliance minimum model, the density filter is universal. Besides, the implementation of density filter is shown as follows.

$$\rho_i = \frac{\sum w_{ij} d_j}{\sum w_{ij}} \tag{6}$$

$$w_{ij} = \begin{cases} \frac{r_0 - r_{ij}}{r_0} & r_{ij} < r_0 \\ 0 & r_{ij} \ge r_0 \end{cases} \tag{7}$$

where d_j denotes the non-filtered element j; w_{ij} denotes the filtering weight of element i and j; r_{ij} denotes the distance of element i and j; r_0 denotes the filter radius.

The interpolation formula of Young's module includes two categories. Such as, the SIMP method, RAMP method.

(1) The SIMP method. The interpolation formula of Young's module is defined as follows:

$$E_e(\rho) = E_{min} + \rho^p (E_0 - E_{min}) \tag{3}$$

Where E_0 denotes the Young's modulus; E_{min} is a very small number, and p is a penalization factor.

(2) The RAMP method. The interpolation formula of Young's module is shown as follows:

$$\frac{1}{E_e(x_e)} = \frac{1}{E_{min}} + x_e\left(\frac{1}{E_0} - \frac{1}{E_{min}}\right) \tag{4}$$

After some transformation, we can obtain:

$$E_e(x_e) = E_{min} + \frac{x_e}{1+q(1-x_e)}(E_0 - E_{min}) \tag{5}$$

The Von-Mises stress which corresponds to the two-dimensional examples with plane stress and bilinear square elements is given as

$$\sigma_{\text{Von-Mises}} = \sqrt{\sigma_x^2 + \sigma_y^2 + \sigma_z^2 + 3\sigma_{\text{xy}}^2} \tag{6}$$

The stress tensor which corresponds to the two-dimensional examples is shown as follows:

$$\boldsymbol{\sigma} = [\sigma_x \quad \sigma_y \quad \sigma_{\text{xy}}]^T = \boldsymbol{DBU} \tag{7}$$

$$\boldsymbol{D} = \frac{E}{1-\nu}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \tag{8}$$

$$\boldsymbol{B} = \frac{1}{2L}\begin{bmatrix} -1 & 0 & 1 & 0 & 1 & 0 & -1 & 0 \\ 0 & -1 & 0 & -1 & 0 & 1 & 0 & 1 \\ -1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 \end{bmatrix} \tag{9}$$

where $\boldsymbol{D}$ denotes the constructive matrix; $\boldsymbol{B}$ denotes the shape function matrix; ν denotes the Poisson ratio; L denotes the element edge length.

3. Cantilever beam

A cantilever beam shown in Fig.1 is investigated in this section. The left surface is fixed, and the force is exerted on the center of right surface.

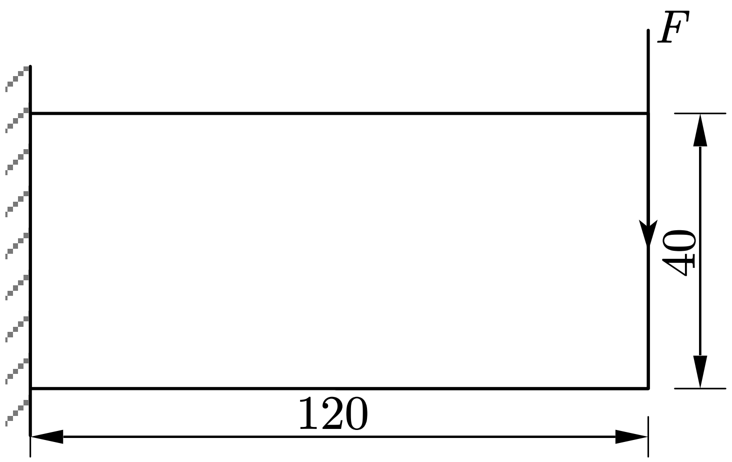


Fig. 1 A cantilever beam

Herein, the number of nodes is 120×40, the external force is 1 and it exerts on element 3; The Poisson's ratio is 0.3, and the radium of filter is 1.7. If the objective function is smallest volume and constraint condition is Von-Mises stress, the corresponding results are shown as Table 1.

Some conclusion can be drawn from Table 1:

(i) The iteration number of the RAMP method is larger than the SIMP method. Therefore, the convergence speed of the SIMP method is faster.

(ii) The volume of the RAMP method is smaller than the SIMP method, but the compliance of the RAMP method is larger than the SIMP method.

(iii) During the iterative process, the Von-Mises stress of the RAMP method is stable, but that of the SIMP method fluctuates greatly.

Herein, the number of nodes is 120×40, the external force is 1 and it exerts on element 3. The Poisson's ratio is 0.3, and the radium of filter is 1.2. If the objective function and constraint condition are compliance and volume respectively, the corresponding results are shown as Table 2.

Table 1. The contrast between the SIMP method and the RAMP method

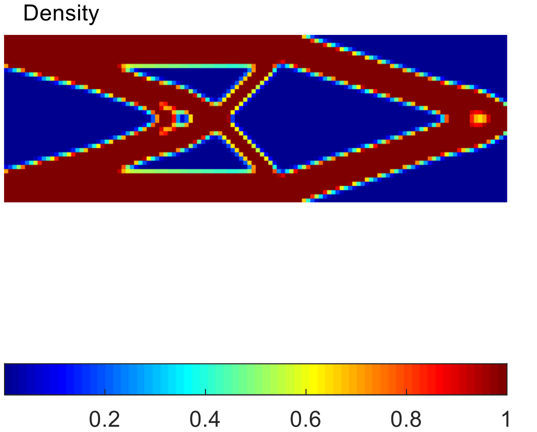
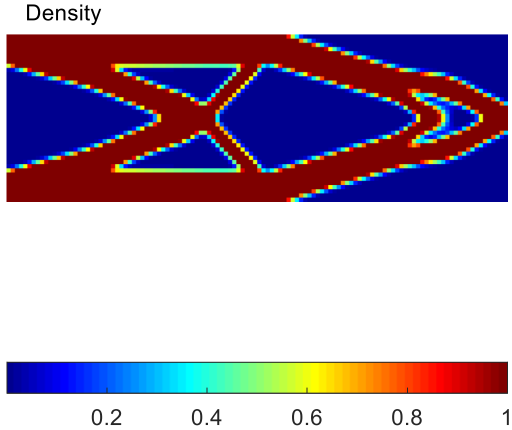
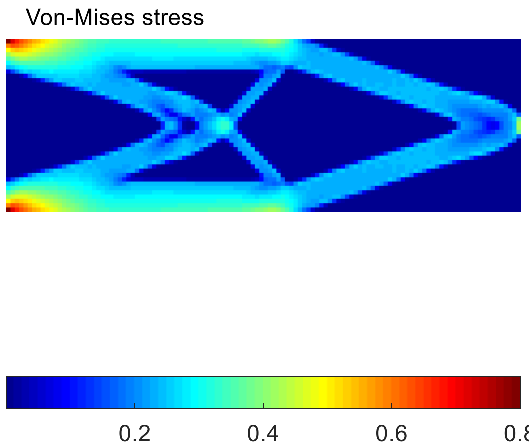
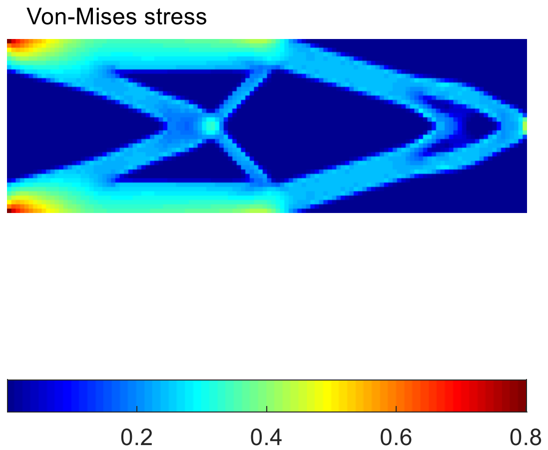
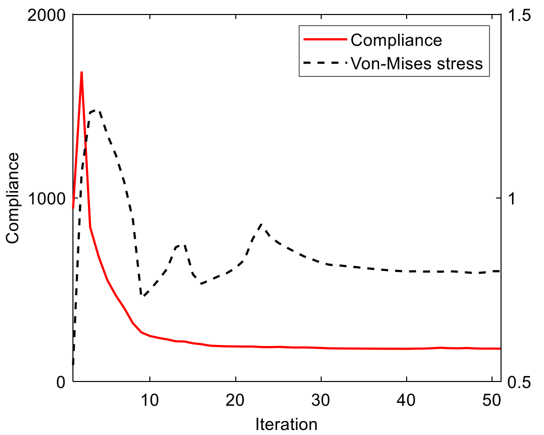
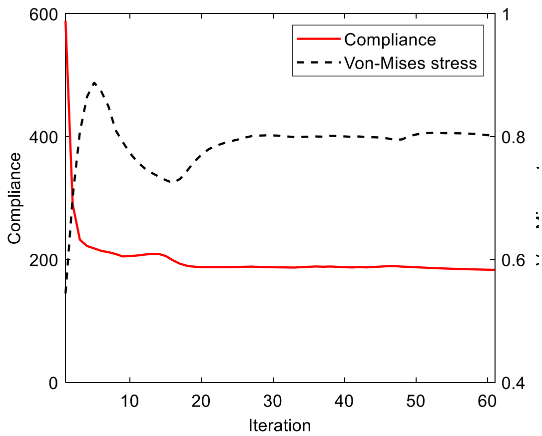
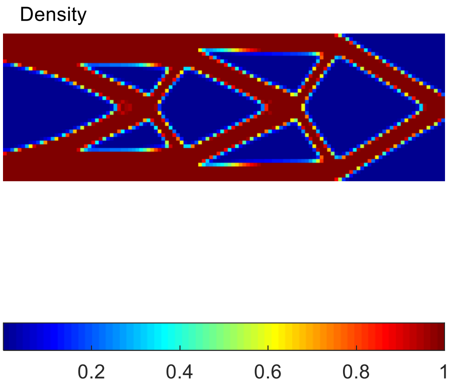
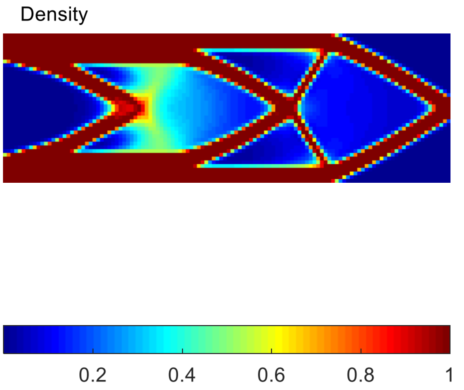
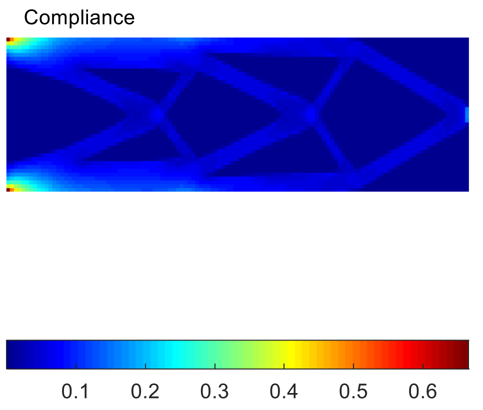
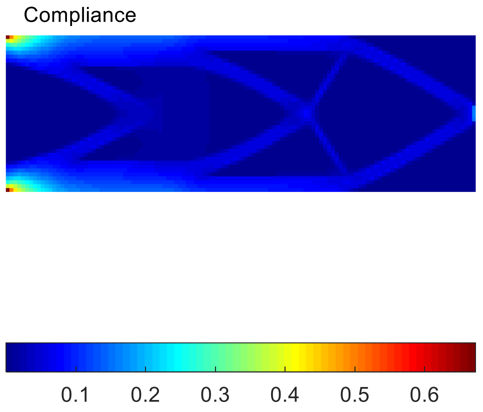
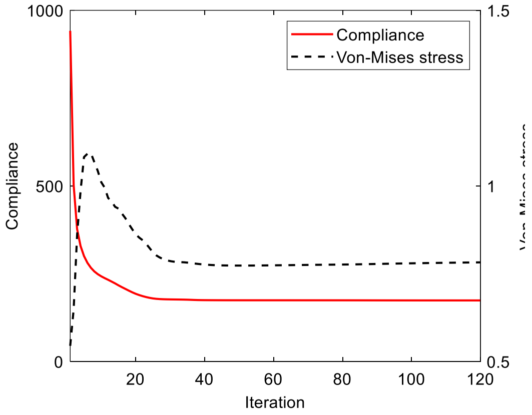
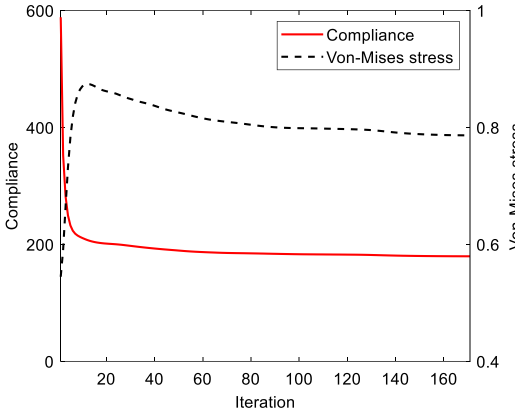
	SIMP	RAMP
Density		
Stress		
Iteration curves		
Result	Total iteration: 51 Volume: 0.52 Compliance: 178.61 Von-Mises stress: 0.8	Total iteration:61 Volume: 0.50 Compliance: 183.17 Von-Mises stress: 0.8

Table 2. Shows the contrast between the SIMP method and the RAMP method.

	SIMP	RAMP
Density		
Compliance		
Iteration curves		
Result	Total iteration: 120 Volume: 0.50 Compliance: 173.91 Von-Mises stress: 0.78	Total iteration:171 Volume: 0.50 Compliance: 179.64 Von-Mises stress: 0.79

Some conclusion can be drawn from Table 2:

(i) The iteration number of the RAMP method is larger than the SIMP method. Therefore, the convergence speed of the SIMP method is faster.

(ii) The Von-Mises stress obtained by the RAMP method is larger than the SIMP method, and the compliance computed by the RAMP method is larger too.

(iii) During the iterative process, the Von-Mises stress of the RAMP method is stable, but that of the SIMP method fluctuates greatly.

(iv) The structure obtained by the RAMP method has numerous gray elements with density of 0.5.

4. Conclusion

The Young's modulus is calculated by the SIMP method or the RAMP method, and the topology optimization model is solved by the proportional topology optimization method. The computational results obtained by two kinds of variable density methods are compared with each other. Some conclusion can be drawn as follows:

(i) The iteration number of the RAMP method is larger than the SIMP method. If the objective function is the smallest volume, the RAMP method can obtain a structure with smaller volume than the SIMP method. However, if the objective function is smallest compliance, the structure obtained by the RAMP method is inferior to that of the SIMP method.

(ii) If the objective function is smallest volume, the structure obtained by the RAMP method is better than that of SIMP method; if the objective function is smallest compliance, the structure obtained by the SIMP method is better than the RAMP method.

(iii) The iteration curve of the RAMP method is much better than the SIMP method.

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