Comparative Analysis of Optimal Strategies under Different Game Power Structures in Complementary Product Supply Chain

Fengshan Si, Tongtong Ge, Jing Wang

School of Management Science and Engineering, Anhui University of Finance and Economics, Bengbu 233030, China

Abstract

The supply chain system is composed of two manufacturers and a retailer. The products produced by the two manufacturers are complementary, and the retailer is responsible for selling the two products. This paper analyzes the four kinds of game power structures in the supply chain, and gives the analytical expressions of the optimal wholesale price and the optimal retail price of the product, and explores the influence of the production cost of the unit product on the optimal strategy. The research shows that complementary products must reduce the manufacturing cost at the same time in order to make both manufacturers profit; The dominant manufacturers make more profits; Retailers are in a weak position in the game, and the peer-to-peer game between two manufacturers is more beneficial to retailers.

Keywords

Supply Chain; Complementary Products; Game Power; Optimal Strategy.

1. Introduction

At present, most of the existing literatures focus on the optimal strategies under a certain game power structure in the supply chain, such as centralized decision [1], decentralized decision [2,3], Stackelberg game [4,5], etc. However, for the supply chain that produces and sells complementary products, it is extremely rare to study the optimal strategies under the four game power structures, especially the comparative analysis of the impact of the production cost of unit product on the optimal strategies. Therefore, this paper will carry out research on this, and strive to provide reference for manufacturers and retailers to make decisions.

2. The Model

The supply chain structure studied in this paper is shown in Fig. 1. It consists of manufacturer 1, manufacturer 2 and a retailer. Manufacturer 1 and manufacturer 2 produce product 1 and product 2 respectively. The two products are complementary. For example, the sales of product 1 will drive the sales of product 2, and vice versa.

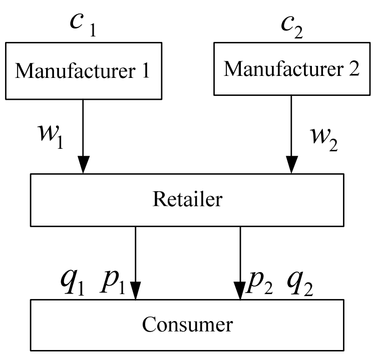

Fig 1. Supply chain structure

The parameter symbols involved in this paper are described as follows:

Table 1. Description of parameter symbols

Symbol	Description	Symbol	Description
c_1	Production cost per unit product 1	c_2	Production cost per unit product 2
w_1	Wholesale price of product 1	w_2	Wholesale price of product 2
q_1	Sales quantity of product 1	q_2	Sales quantity of product 2
p_1	Retail price of product 1	p_2	Retail price of product 2
a_1	Potential maximum market demand of product 1	a_2	Potential maximum market demand of product 2
α	Sensitivity coefficient of product sales quantity to price	β	Complementary coefficient between product 1 and product 2
π_{m1}	Profit of manufacturer 1	π_{m2}	Profit of manufacturer 2
π_r	Retailer's profit	*	Optimal strategy

For the convenience of analysis, the following assumptions and explanations are made:

(1) Only the production cost is considered, and the sales cost is not considered.

(2) In order to ensure that manufacturers and retailer can make profits, it is required that $p_i > w_i > c_i > 0$, $i = 1,2$.

(3) The impact of the retail price of the product on the sales quantity is higher than that of the complementary product. Therefore, it is necessary to ensure that $\alpha > \beta > 0$.

The demand functions of product 1 and product 2 are:

$$q_1 = a_1 - \alpha p_1 - \beta p_2 \tag{1}$$

$$q_2 = a_2 - \alpha p_2 - \beta p_1 \tag{2}$$

The profit functions of manufacturers and retailer are:

$$\pi_{m1}(w_1) = (w_1 - c_1)q_1 \tag{3}$$

$$\pi_{m2}(w_2) = (w_2 - c_2)q_2 \tag{4}$$

$$\pi_r(p_1, p_2) = (p_1 - w_1)q_1 + (p_2 - w_2)q_2 \tag{5}$$

3. Stackelberg Game

Manufacturer 1 and manufacturer 2 are leaders and the retailer is follower. Manufacturers formulate the wholesale price first, and the retailer formulates the retail price on this basis. According to the reverse algorithm, the optimal retail price is obtained first.

$$\frac{\partial^2 \pi_r(p_1, p_2)}{\partial p_1^2} = \frac{\partial^2 \pi_r(p_1, p_2)}{\partial p_2^2} = -2\alpha \, , \; \frac{\partial^2 \pi_r(p_1, p_2)}{\partial p_1 p_2} = \frac{\partial^2 \pi_r(p_1, p_2)}{\partial p_2 p_1} = -2\beta$$

The Hessian matrix is $\begin{bmatrix} -2\alpha, -2\beta \\ -2\beta, -2\alpha \end{bmatrix}$. The odd order principal sub formula of the Hessian matrix is negative, and the even order principal sub formula is positive. The Hessian matrix is a negative definite matrix, that is, there is an optimal retail price to maximize the profit of the retailer. Simultaneous $\frac{\partial \pi_r(p_1,p_2)}{\partial p_1}$=0 and $\frac{\partial \pi_r(p_1,p_2)}{\partial p_2}$=0 can be obtained:

$$p_1^*(w_1)=\frac{\alpha a_1-\beta a_2+\alpha^2 w_1-\beta^2 w_1}{2\left(\alpha^2-\beta^2\right)} \tag{6}$$

$$p_2^*(w_2)=\frac{\beta a_1+\alpha a_2+\alpha^2 w_2-\beta^2 w_2}{2\left(\alpha^2-\beta^2\right)} \tag{7}$$

3.1. Peer-to-Peer Game between Manufacturer 1 and Manufacturer 2

Substituting Eq. (6) and Eq.(7) into Eq. (3) and Eq. (4) respectively, simultaneous $\frac{\partial \pi_{m1}(w_1)}{\partial w_1}$=0 and $\frac{\partial \pi_{m2}(w_2)}{\partial w_2}$=0 can be get:

$$w_1^*=\frac{2\alpha a_1+2\alpha^2 c_1-\beta\left(a_2+\alpha c_2\right)}{4\alpha^2-\beta^2} \tag{8}$$

$$w_2^*=\frac{\beta a_1+\alpha\left(\beta c_1-2a_2-2\alpha c_2\right)}{\beta^2-4\alpha^2} \tag{9}$$

Substituting Eq. (8) and Eq. (9) into Eq. (6) and Eq. (7) to obtain:

$$p_1^*=\frac{\left(6\alpha^3-3\alpha\beta^2\right)a_1+\left(2\beta^3-5\alpha^2\beta\right)a_2+\alpha\left(\alpha^2-\beta^2\right)\left(2\alpha c_1-\beta c_2\right)}{2\left(4\alpha^4-5\alpha^2\beta^2+\beta^4\right)} \tag{10}$$

$$p_2^*=\frac{\left(2\beta^3-5\alpha^2\beta\right)a_1+\alpha\left(6\alpha^2-3\beta^2\right)a_2+\alpha\left(\alpha^2-\beta^2\right)\left(2\alpha c_2-\beta c_1\right)}{2\left(4\alpha^4-5\alpha^2\beta^2+\beta^4\right)} \tag{11}$$

By bring Eqs. (8) - (11) into Eqs. (1) - (5), the optimal profit of manufacturers and retailer can be obtained. The following analysis is similar to that. The optimal profits of manufacturers and retailer can be obtained by the optimal strategy, so I won't repeat it again.

3.2. Manufacturer 1 is the Leader and Manufacturer 2 is the Follower

Substituting Eq. (6) and Eq. (7) into Eq. (4) to obtain $\frac{\partial^2 \pi_{m2}(w_2)}{\partial w_2^2}=-\alpha<0$. Therefore, there is an optimal wholesale price w_2 to maximize the profit of manufacturer 2. Solve $\frac{\partial \pi_{m2}(w_2)}{\partial w_2}$=0 to obtain:

$$w_2^*=\frac{a_2+\alpha c_2-\beta w_1}{2\alpha} \tag{12}$$

Eq. (12) is substituted into Eq. (7) to obtain:

$$p_2^*=\frac{\left(3\alpha^2-\beta^2\right)a_2+\left(\alpha^2-\beta^2\right)\left(\alpha c_2-\beta w_1\right)-2\alpha\beta a_1}{4\left(\alpha^3-\alpha\beta^2\right)} \tag{13}$$

Substituting Eq. (6) and Eq. (13) into Eq. (3) to obtain $\frac{\partial^2\pi_{m1}(w_1)}{\partial w_1^2}=\frac{\beta^2-2\alpha^2}{2\alpha}<0$. Therefore, there is an optimal wholesale price w_1 to maximize the profit of manufacturer 1. Solve $\frac{\partial\pi_{m1}(w_1)}{\partial w_1}$=0 to obtain:

$$w_1^*=\frac{2\alpha a_1+2\alpha^2c_1-\beta\left(a_2+\beta c_1+\alpha c_2\right)}{4\alpha^2-2\beta^2} \tag{14}$$

Substituting Eq. (14) into Eq. (6), Eq. (12) and Eq. (13) respectively

$$p_1^*=\frac{\left(6\alpha^3-4\alpha\beta^2\right)a_1+\left(3\beta^3-5\alpha^2\beta\right)a_2+\left(\alpha^2-\beta^2\right)\left(\left(2\alpha^2-\beta^2\right)c_1-\alpha\beta c_2\right)}{4\left(2\alpha^4-3\alpha^2\beta^2+\beta^4\right)} \tag{15}$$

$$w_2^*=\frac{\left(4\alpha^2-\beta^2\right)a_2-2\alpha^2\beta c_1+\beta^3c_1+4\alpha^3c_2-\alpha\beta^2c_2-2\alpha\beta a_1}{8\alpha^3-4\alpha\beta^2} \tag{16}$$

$$p_2^*=\frac{\left(6\alpha\beta^3-10\alpha^3\beta\right)a_1+\left(12\alpha^4-9\alpha^2\beta^2+\beta^4\right)a_2+\left(\alpha^2-\beta^2\right)\left(\left(\beta^3-2\alpha^2\beta\right)c_1+\alpha\left(4\alpha^2-\beta^2\right)c_2\right)}{8\alpha\left(2\alpha^4-3\alpha^2\beta^2+\beta^4\right)} \tag{17}$$

3.3. Manufacturer 2 is the Leader and Manufacturer 1 is the Follower

Substituting Eq. (6) and Eq. (7) into Eq. (3) to obtain $\frac{\partial^2\pi_{m1}(w_1)}{\partial w_1^2}=-\alpha<0$. Therefore, there is an optimal wholesale price w_1 to maximize the profit of manufacturer 1. Calculate $\frac{\partial\pi_{m1}(w_1)}{\partial w_1}$=0 to get:

$$w_1^*=\frac{a_1+\alpha c_1-\beta w_2}{2\alpha} \tag{18}$$

Substituting Eq. (18) into Eq. (6) to obtain:

$$p_1^*=\frac{\left(3\alpha^2-\beta^2\right)a_1-2\alpha\beta a_2+\left(\alpha^2-\beta^2\right)\left(\alpha c_1-\beta w_2\right)}{4\left(\alpha^3-\alpha\beta^2\right)} \tag{19}$$

Substituting Eq. (7) and Eq. (19) into Eq. (4) to obtain $\frac{\partial^2\pi_{m2}(w_2)}{\partial w_2^2}=\frac{\beta^2-2\alpha^2}{2\alpha}<0$. Therefore, there is an optimal wholesale price w_2 to maximize the profit of manufacturer 2. Calculate $\frac{\partial\pi_{m2}(w_2)}{\partial w_2}$=0 to get:

$$w_2^*=\frac{2\alpha a_2-\alpha\beta c_1+2\alpha^2c_2-\beta^2c_2-\beta a_1}{2\left(2\alpha^2-\beta^2\right)} \tag{20}$$

Substituting Eq. (20) into Eq. (7), Eq. (18) and Eq. (19) respectively

$$p_2^* = \frac{\left(3\beta^3 - 5\alpha^2\beta\right)a_1 + \left(6\alpha^3 - 4\alpha\beta^2\right)a_2 + \left(\alpha^2 - \beta^2\right)\left(\left(2\alpha^2 - \beta^2\right)c_2 - \alpha\beta c_1\right)}{4\left(2\alpha^4 - 3\alpha^2\beta^2 + \beta^4\right)} \tag{21}$$

$$w_1^* = \frac{\left(4\alpha^2 - \beta^2\right)a_1 - 2\alpha\beta a_2 + 4\alpha^3 c_1 - \alpha\beta^2 c_1 - 2\alpha^2\beta c_2 + \beta^3 c_2}{8\alpha^3 - 4\alpha\beta^2} \tag{22}$$

$$p_1^* = \frac{\left(12\alpha^4 - 9\alpha^2\beta^2 + \beta^4\right)a_1 + \left(6\alpha\beta^3 - 10\alpha^3\beta\right)a_2 + \left(\alpha^2 - \beta^2\right)\left(\left(4\alpha^3 - \alpha\beta^2\right)c_1 + \beta\left(\beta^2 - 2\alpha^2\right)c_2\right)}{8\alpha\left(2\alpha^4 - 3\alpha^2\beta^2 + \beta^4\right)} \tag{23}$$

4. Nash Game

Manufacturer 1, manufacturer 2 and retailer make decisions at the same time. Manufacturer 1 decides the wholesale price w_1, manufacturer 2 decides the wholesale price w_2, and retailer decides the retail price p_1 and p_2.

It can be seen from the above Hessian matrix analysis that $\pi_r(p_1, p_2)$ is a concave function and there is an optimal retail price to maximize the retailer's profit. Set $p_1 = w_1 + m_1$, $p_2 = w_2 + m_2$, Where m_i is the difference between the retail price and the wholesale price, $m_i \ge 0$, $i = 1, 2$. Substituting p_i into Eq. (1) - (4) to obtain $\frac{\partial^2 \pi_{m1}(w_1)}{\partial w_1^2} = \frac{\partial^2 \pi_{m2}(w_2)}{\partial w_2^2} = -2\alpha < 0$. Therefore, both $\pi_{m1}(w_1)$ and $\pi_{m2}(w_2)$ are concave functions, the existence of the optimal wholesale price maximizes the profit of the manufacturers.

Simultaneous $\frac{\partial \pi_{m1}(w_1)}{\partial w_1} = 0$, $\frac{\partial \pi_{m2}(w_2)}{\partial w_2} = 0$, $\frac{\partial \pi_r(p_1, p_2)}{\partial p_1} = 0$ and $\frac{\partial \pi_r(p_1, p_2)}{\partial p_2} = 0$ to obtain:

$$p_1^* = \frac{\alpha a_1 - \beta a_2}{\alpha^2 - \beta^2} \tag{24}$$

$$p_2^* = \frac{\alpha a_2 - \beta a_1}{\alpha^2 - \beta^2} \tag{25}$$

$$w_1^* = \frac{\alpha a_1 - \beta a_2}{\alpha^2 - \beta^2} \tag{26}$$

$$w_2^* = \frac{\alpha a_2 - \beta a_1}{\alpha^2 - \beta^2} \tag{27}$$

By comparing Eq. (24) and Eq. (26), Eq. (25) and Eq. (27), it can be seen that the retail price of the two products is equal to their wholesale price, which means that the retailer is unprofitable, so the retailer will resist the simultaneous decision. In this case, numerical simulation will not be conducted in Section 5.

5. Numerical Simulation

In order to compare and analyze the influence of the manufacturing cost per unit product on the optimal strategy under different game power structures, on the basis of satisfying the model constraints, a group of parameter values are taken as follows: $a_1 = 5$, $a_2 = 5$, $\alpha = 0.8$, $\beta = 0.5$, $c_1 = 0.2$, $c_2 = 0.3$.

5.1. Optimal Strategies When $c_1 = 0.2 < c_2 = 0.3$

Table 2. Under three game power structures when $c_1 = 0.2 < c_2 = 0.3$

Decision variables	Peer-peer game between manufacturer 1 and manufacturer 2	Manufacturer 1 is the leader, manufacturer 2 is a follower	Manufacturer 2 is the leader, manufacturer 1 is a follower
w_1^*	2.43983	2.71165	2.35592
w_2^*	2.51255	2.42761	2.78107
p_1^*	3.14299	3.27890	3.10104
p_2^*	3.17935	3.13688	3.31361
π_{m1}^*	2.00673	2.03051	1.85919
π_{m2}^*	1.95816	1.81069	1.98137
π_r^*	1.22012	1.06221	1.06785

5.2. Optimal Strategies When $c_1 = c_2 = 0.3$

Table 3. Optimal strategies under three game power structures when $c_1 = c_2 = 0.3$

Decision variables	Peer-peer game between manufacturer 1 and manufacturer 2	Manufacturer 1 is the leader, manufacturer 2 is a follower	Manufacturer 2 is the leader, manufacturer 1 is a follower
w_1^*	2.49524	2.76165	2.41198
w_2^*	2.49524	2.41198	2.76165
p_1^*	3.17070	3.30390	3.12907
p_2^*	3.17070	3.12907	3.30390
π_{m1}^*	1.92763	1.95047	1.78419
π_{m2}^*	1.92763	1.78419	1.95047
π_r^*	1.18623	1.03544	1.03544

5.3. Optimal Strategies When $c_1 = 0.4 > c_2 = 0.3$

Table 4. Optimal strategies under three game power structures when $c_1 = 0.4 > c_2 = 0.3$

Decision variables	Peer-peer game between manufacturer 1 and manufacturer 2	Manufacturer 1 is the leader, manufacturer 2 is a follower	Manufacturer 2 is the leader, manufacturer 1 is a follower
w_1^*	2.55065	2.81165	2.46805
w_2^*	2.47792	2.39636	2.74223
p_1^*	3.19840	3.32890	3.15710
p_2^*	3.16204	3.12126	3.29419
π_{m1}^*	1.85012	1.87204	1.71074
π_{m2}^*	1.89734	1.75789	1.91982
π_r^*	1.15322	1.00937	1.00389

Comparing Table 2, Table 3 and Table 4, it is found that (1) no matter what kind of game power structure, the profit of manufacturer 1, manufacturer 2 and retailer will decrease with the increase of manufacturing cost per unit product of manufacturer 1; The wholesale price and retail price of product 1 both increased, while the wholesale price and retail price of product 2 both decreased. (2) No matter what kind of game power structure, no matter how the unit product production cost changes, the retailer's profit is the lowest among the three game players. (3) In the peer-to-peer game structure of manufacturer 1 and manufacturer 2, with the increase of unit production cost of product 1, the profit of manufacturer 1 decreases the most, and is gradually lower than that of manufacturer 2. (4) In the case of unequal game power structure between manufacturer 1 and manufacturer 2, no matter how the manufacturing cost per unit product changes, the manufacturer as the leader always gains the most. (5) The retailer gains the most when two manufacturers peer-to-peer game.

6. Conclusion

By constructing a two-level supply chain model composed of manufacturers and a retailer, this paper analyzes the optimal strategies under four kinds of game structures, and gives an analytical formula for the optimal strategies. Through numerical simulation, the influence of production cost per unit product on the optimal strategy is explored. The management enlightenment obtained through comparative analysis is as follows: (1) for the two complementary products, one party's increase in the production cost of unit product will lead to the reduction of the profits of both parties, so reducing the production cost of unit product at the same time can promote both parties to increase profits. (2) In the game with manufacturers, retailers are in a disadvantaged position. In order to obtain more profits, they must take additional measures, such as improving service levels and striving for subsidies from manufacturers. (3) In order to make the most profit, the two manufacturers do not favor the structure of equal power in the game. They will try their best to break the balance and strive for more power to speak, so as to make themselves the leaders in the rule making. However, retailers prefer the emergence of the phenomenon of two manufacturers' equal game. This paper will further study the complementary coefficient between products in the future.

Acknowledgments

Scientific Research Project of Anhui University of Finance and Economics (ACKYB22015).

References

[1] P.Chintapalli, S. M. Disney, C. S. Tang. Coordinating Supply Chains via Advance-Order Discounts, Minimum Order Quantities, and Delegations[J]. Production and Operations Management Society, 2017, 26(12): 2175-2186.

[2] J. T. Xu, Q. Qi, Q. G. Bai. Coordinating a dual-channel supply chain with price discount contracts under carbon emission capacity regulation[J]. Applied Mathematical Modelling, 2018, 56(4): 449-468.

[3] H. H. Song, X. X. Gao. Green supply chain game model and analysis under revenue-sharing Contract[J]. Journal of Cleaner Production, 2018, 170(1):183-192.

[4] Y. Q. Lan, Y. Z. Li, F. Papier. Competition and coordination in a three-tier supply chain with differentiated channels[J]. European Journal of Operational Research, 2018, 269(3):870-882.

[5] J. Zhao, X. R. Hou, Y. L. Guo, J. Wei. Pricing policies for complementary products in a dual-channel supply chain[J]. Applied Mathematical Modelling, 2017, 49(9):437-451.